Second order polynomial Hamiltonian systems with $\tilde{W}(E_6^{(1)}), \tilde{W}(E_7^{(1)})$ and $W(E_8^{(1)})$-symmetry

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Abstract

We find and study a six (resp. seven, eight)-parameter family of polynomial Hamiltonian systems of second order, respectively. This system admits the affine Weyl group symmetry of type $E_6^{(1)}$ (resp. $E_7^{(1)}, E_8^{(1)}$) as the group of its Bäcklund transformations. Each system is the first example which gave second-order polynomial Hamiltonian system with $\tilde{W}(E_6^{(1)})$ (resp. $\tilde{W}(E_7^{(1)}), W(E_8^{(1)})$)-symmetry. We also show that its space of initial conditions S is obtained by gluing eight (resp. nine, ten) copies of $\mathbb{C}^2$ via the birational and symplectic transformations.

Key Words and Phrases. Affine Weyl group, Bäcklund transformation, Holomorphy condition, Painlevé equations.

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1 Main results of the system with $\tilde{W}(E_6^{(1)})$-symmetry

In this paper, we find and study a 6-parameter family of polynomial Hamiltonian systems of second order. This system admits extended affine Weyl group symmetry of type $E_6^{(1)}$ (see Figure 1) as the group of its Bäcklund transformations. This system is the first example which gave second-order polynomial Hamiltonian system with $\tilde{W}(E_6^{(1)})$-symmetry.

By eliminating p or q, we obtain the second-order ordinary differential equation. However, its form is not normal (cf. [8, 9]).

We also show that after a series of explicit blowing-ups at nine points including the infinitely near points of the Hirzebruch surface Σ_2 (see Figure 2) and blowing-down along the (-1)-curve $H' \cong \mathbb{P}^1$ to a nonsingular point (see Figure 3), we obtain the rational surface $\tilde{S}$ and a birational morphism

$$\varphi : \tilde{S} \leftarrow S_9 \rightarrow \cdots \rightarrow S_1 \rightarrow \Sigma_2.$$

Here, the symbol H' denotes the strict transform of H, each E_i denotes the exceptional divisors, and $-K_{\Sigma_2} = 2H,\ H \cong \mathbb{P}^1,\ (H)^2 = 2$. In order to obtain a minimal compactification of the space of initial conditions, we must blow down along the (-1)-curve H'.

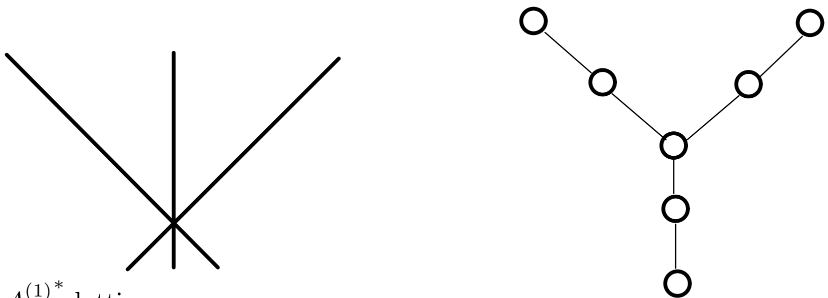
Its canonical divisor $K_{\tilde{S}}$ of $\tilde{S}$ is given by

$$K_{\tilde{S}} = -\sum_{i=1}^{3} E_i, \quad (E_i)^2 = -2, \quad E_i \cong \mathbb{P}^1, \quad E_1 \cap E_2 \cap E_3 \neq \varnothing, \quad (E_j, E_k) = 1 \quad (j \neq k). \tag{1}$$

This configuration of $(-K_{\tilde{S}})_{red}$ is type ${A_2^{(1)}}^*$ (see Figure 1). This type of rational surface $\tilde{S}$ does not appear in the list of Painlevé equations (see [7]).

The space of initial conditions S is obtained by gluing eight copies of $\mathbb{C}^2$

$$\begin{aligned} S &= \tilde{S} - (-K_{\tilde{S}})_{red} \\ &= \mathbb{C}^2 \cup \bigcup_{i=0}^{6} \mathbb{C}^2 \end{aligned} \tag{2}$$



$A_2^{(1)^*}$-lattice

Dynkin diagram of type $E_6^{(1)}$

Figure 1: $A_2^{(1)^*}$-lattice and Dynkin diagram of type $E_6^{(1)}$

via the birational and symplectic transformations r_j (see Theorem 2.1).

Painlevé equations	PVI	PV	PIV	$PIII^{D_6^{(1)}}$	$PIII^{D_7^{(1)}}$	$PIII^{D_8^{(1)}}$	PII	PI
Type of surface	$D_4^{(1)}$	$D_5^{(1)}$	$E_6^{(1)}$	$D_6^{(1)}$	$D_7^{(1)}$	$D_8^{(1)}$	$E_7^{(1)}$	$E_8^{(1)}$
Symmetry	$D_4^{(1)}$	$A_3^{(1)}$	$A_2^{(1)}$	$C_2^{(1)}$	$A_1^{(1)}$	none	$A_1^{(1)}$	none

This system is the first example whose minimal model $\tilde{S}$ is the rational surface of type $A_2^{(1)^*}$.
The author believes that this system can be obtained by holonomic deformation of the 3rd-order linear ordinary differential equation

$$\frac{d^3y}{dx^3}+a_1(x)\frac{d^2y}{dx^2}+a_2(x)\frac{dy}{dx}+a_3(x)y=0 \quad (a_i \in \mathbb{C}(x)) \tag{3}$$

satisfying the Riemann scheme:

$$\begin{pmatrix} x=0 & x=\frac{1}{\varepsilon} & x=q & x=\infty \\ 0 & 0 & 0 & \alpha_0 \\ \alpha_2 & \alpha_4 & 1 & \alpha_0+\alpha_5 \\ \alpha_1+\alpha_2 & \alpha_3+\alpha_4 & 3 & \alpha_0+\alpha_5+\alpha_6 \end{pmatrix}, \tag{4}$$

where $x=q$ is an apparent singular point. In this case, $\varepsilon=1$.
It is still an open question whether the equation (30) satisfying (31) tends to the equation (30) satisfying the Riemann scheme:

$$\begin{pmatrix} x=0 & x=q & x=\infty & x=\infty \\ 0 & 0 & 0 & \alpha_4 \\ \alpha_2 & 1 & 1 & \alpha_3 \\ \alpha_1+\alpha_2 & 3 & t & \alpha_0 \end{pmatrix} \tag{5}$$

as $\varepsilon \to 0$, where $x=\infty$ is an irregular singular point with Poincaré rank 1 (cf. [12, 11]).

2 Holomorphy

Theorem 2.1. *Let us consider a polynomial Hamiltonian system with Hamiltonian $I \in \mathbb{C}(t)[q,p]$. We assume that*
(A1) $deg(I)=7$ with respect to q,p.
(A2) This system becomes again a polynomial Hamiltonian system in each coordinate r_i $(i=0,1,2,3,4)$:

$$\begin{aligned}
&r_0: x_0=1/q,\ y_0=-(qp+\alpha_0)q,\\
&r_1: x_1=-(pq-(\alpha_1+\alpha_2))p,\ y_1=1/p,\\
&r_2: x_2=-(pq-\alpha_2)p,\ y_2=1/p,\\
&r_3: x_3=-(p(q-1)-(\alpha_3+\alpha_4))p,\ y_3=1/p,\\
&r_4: x_4=-(p(q-1)-\alpha_4)p,\ y_4=1/p.
\end{aligned}$$

(A3) In addition to the assumption (A2), the Hamiltonian system in the coordinate r_0 becomes again a polynomial Hamiltonian system in the coordinates r_5, r_6:

$$\begin{aligned} r_5 &: x_5 = -(x_0y_0 - \alpha_5)y_0, \ y_5 = 1/y_0, \\ r_6 &: x_6 = -(x_0y_0 - (\alpha_5 + \alpha_6))y_0, \ y_6 = 1/y_0. \end{aligned}$$

Then such a system coincides with the system

$$\begin{aligned} &\frac{dq}{dt} = \frac{\partial I}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial I}{\partial q}, \\ I :=&(q-1)^2q^2p^3 - q(q-1)\{(\alpha_1 + 2\alpha_2 + \alpha_3 + 2\alpha_4)q - \alpha_1 - 2\alpha_2\}p^2 \\ &+ [\{-3\alpha_0^2 - 2\alpha_0(\alpha_1 + 2\alpha_2 + \alpha_3 + 2\alpha_4) - 3\alpha_0\alpha_5 - \alpha_5(\alpha_1 + 2\alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5)\}q^2 \\ &\{-3\alpha_0^2 - \alpha_2^2 + 2\alpha_0(\alpha_1 + 2\alpha_2 + \alpha_3 + 2\alpha_4) + 3\alpha_0\alpha_5 + 2\alpha_2\alpha_5 + \alpha_1(\alpha_5 - \alpha_2) \\ &+ (\alpha_4 + \alpha_5)(\alpha_3 + \alpha_4 + \alpha_5)\}q + \alpha_2(\alpha_1 + \alpha_2)]p + \alpha_0(\alpha_0 + \alpha_5)(\alpha_0 + \alpha_5 + \alpha_6)q, \end{aligned} \tag{6}$$

where the constant parameters α_i satisfy the relation:

$$3\alpha_0 + \alpha_1 + 2\alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 + \alpha_6 = 0. \tag{7}$$

Since each transformation r_i is symplectic, the system (6) is transformed into a Hamiltonian system, whose Hamiltonian may have poles. It is remarkable that the transformed system becomes again a polynomial system for any $i = 0, 1, \dots, 6$.
The holomorphy conditions $(A2), (A3)$ are new. Theorem 2.1 can be checked by a direct calculation.

Proposition 2.1. *The Hamiltonian I is its first integral.*

Remark 2.1. *For the Hamiltonian system in each coordinate system (x_i, y_i) $(i = 0, 1, \dots, 6)$ given by (A2) and (A3) in Theorem 2.1, by eliminating x_i or y_i, we obtain the second-order ordinary differential equation. However, its form is not normal* (cf. [8, 9]).

3 Symmetry

Theorem 3.1. *The system (6) admits extended affine Weyl group symmetry of type $E_6^{(1)}$ as the group of its Bäcklund transformations whose generators s_i, $i = 0, 1, \dots, 6$, π_j, $j = 1, 2, 3$ are explicitly given as follows: with the notation $(*) := (q, p, t; \alpha_0, \alpha_1, \dots, \alpha_6)$,*

$$\begin{aligned} s_0 &: (*) \to \left(q + \frac{\alpha_0}{p}, p, t; -\alpha_0, \alpha_1, \alpha_0 + \alpha_2, \alpha_3, \alpha_4 + \alpha_0, \alpha_5 + \alpha_0, \alpha_6\right), \\ s_1 &: (*) \to (q, p, t; \alpha_0, -\alpha_1, \alpha_2 + \alpha_1, \alpha_3, \alpha_4, \alpha_5, \alpha_6), \\ s_2 &: (*) \to \left(q, p - \frac{\alpha_2}{q}, t; \alpha_0 + \alpha_2, \alpha_1 + \alpha_2, -\alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\right), \\ s_3 &: (*) \to (q, p, t; \alpha_0, \alpha_1, \alpha_2, -\alpha_3, \alpha_4 + \alpha_3, \alpha_5, \alpha_6), \\ s_4 &: (*) \to \left(q, p - \frac{\alpha_4}{q-1}, t; \alpha_0 + \alpha_4, \alpha_1, \alpha_2, \alpha_3 + \alpha_4, -\alpha_4, \alpha_5, \alpha_6\right), \\ s_5 &: (*) \to (q, p, t; \alpha_0 + \alpha_5, \alpha_1, \alpha_2, \alpha_3, \alpha_4, -\alpha_5, \alpha_6 + \alpha_5), \\ s_6 &: (*) \to (q, p, t; \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 + \alpha_6, -\alpha_6), \\ \pi_1 &: (*) \to (1 - q, -p, 1 - t; \alpha_0, \alpha_3, \alpha_4, \alpha_1, \alpha_2, \alpha_5, \alpha_6), \\ \pi_2 &: (*) \to \left(\frac{1}{q}, -(qp + \alpha_0)q, -t; \alpha_0, \alpha_6, \alpha_5, \alpha_3, \alpha_4, \alpha_2, \alpha_1\right), \\ \pi_3 &: (*) \to \left(\frac{q}{q-1}, -(q-1)((q-1)p + \alpha_0), 2 - t; \alpha_0, \alpha_1, \alpha_2, \alpha_6, \alpha_5, \alpha_4, \alpha_3\right), \end{aligned}$$

where π_j, $j = 1, 2, 3$ are Dynkin diagram automorphisms of type $E_6^{(1)}$.

Theorem 3.1 can be checked by a direct calculation.

4 Space of initial conditions

Theorem 4.1. *After a series of explicit blowing-ups at nine points including the infinitely near points of Σ_2 and blowing-down along the (-1)-curve $D^{(0)'} \cong \mathbb{P}^1$, we obtain the rational surface $\tilde{S}$ of the system* (6) *and a birational morphism $\varphi : \tilde{S} \cdots \to \Sigma_2$. Its canonical divisor $K_{\tilde{S}}$ of $\tilde{S}$ is given by*

$$K_{\tilde{S}} = -D_0^{(1)} - D_1^{(1)} - D_\infty^{(1)}, \quad (D_\nu^{(1)})^2 = -2, \ D_\nu^{(1)} \cong \mathbb{P}^1, \tag{8}$$

where the symbol $D^{(0)'}$ denotes the strict transform of $D^{(0)}$, $D_\nu^{(1)}$ denote the exceptional divisors and $-K_{\Sigma_2} = 2D^{(0)}$, $D^{(0)} \cong \mathbb{P}^1$, $(D^{(0)})^2 = 2$.

Theorem 4.2. *The space of initial conditions S of the system* (6) *is obtained by gluing eight copies of $\mathbb{C}^2$:*

$$\begin{aligned} S &= \tilde{S} - (-K_{\tilde{S}})_{red} \\ &= \mathbb{C}^2 \cup \bigcup_{i=0}^{6} U_j, \\ \mathbb{C}^2 &\ni (q,p), \quad U_j \cong \mathbb{C}^2 \ni (x_j, y_j) \ (j = 0, 1, \dots, 6) \end{aligned} \tag{9}$$

via the birational and symplectic transformations r_j (see Theorem 2.1).

Proof of Theorems 4.1 and 4.2.

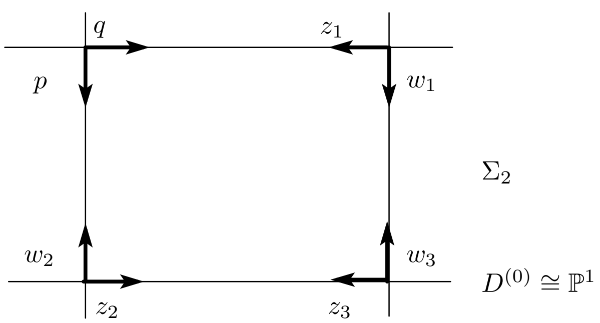


Figure 2: Hirzebruch surface Σ_2

At first, we take the Hirzebruch surface Σ_2 which is obtained by gluing four copies of $\mathbb{C}^2$ via the following identification.

$$\begin{aligned} &U_j \cong \mathbb{C}^2 \ni (z_j, w_j) \ (j = 0, 1, 2, 3) \\ &z_0 = q, \ w_0 = p, \quad z_1 = \frac{1}{q}, \ w_1 = -(qp + \alpha_0)q, \\ &z_2 = z_0, \ w_2 = \frac{1}{w_0}, \quad z_3 = z_1, \ w_3 = \frac{1}{w_1}. \end{aligned} \tag{10}$$

We define a divisor $D^{(0)}$ on Σ_2:

$$D^{(0)} = \{(z_2, w_2) \in U_2 | w_2 = 0\} \cup \{(z_3, w_3) \in U_3 | w_3 = 0\} \cong \mathbb{P}^1. \tag{11}$$

The self-intersection number of $D^{(0)}$ is given by

$$(D^{(0)})^2 = 2. \tag{12}$$

By a direct calculation, we see that the system (6) has three accessible singular points $a_\nu^{(0)} \in D^{(0)}$ $(\nu = 0, 1, \infty)$:

$$\begin{aligned} a_\nu^{(0)} &= \{(z_2, w_2) = (\nu, 0)\} \in U_2 \cap D^{(0)} \ (\nu = 0, 1), \\ a_\infty^{(0)} &= \{(z_3, w_3) = (0, 0)\} \in U_3 \cap D^{(0)}. \end{aligned} \tag{13}$$

We perform blowing-ups in Σ_2 at $a_\nu^{(0)}$, and let $D_\nu^{(1)}$ be the exceptional curves of the blowing-ups at $a_\nu^{(0)}$ for $\nu = 0, 1, \infty$. We can take three coordinate systems (u_ν, v_ν) around the points at infinity of the exceptional curves $D_\nu^{(1)} \quad (\nu = 0, 1, \infty)$, where

$$\begin{aligned}
(u_\nu, v_\nu) &= \left(\frac{z_2 - \nu}{w_2}, w_2\right) \ (\nu = 0, 1), \\
(u_\infty, v_\infty) &= \left(\frac{z_3}{w_3}, w_3\right).
\end{aligned} \tag{14}$$

Note that $\{(u_\nu, v_\nu) | v_\nu = 0\} \subset D_\nu^{(1)}$ for $\nu = 0, 1, \infty$. By a direct calculation, we see that the system (54) has six accessible singular points $a_\nu^{(1)}$ for $\nu = 1, 2, 3, 4, 5, 6$ in $D_\nu^{(1)} \cong \mathbb{P}^1$ $(\nu = 0, 1, \infty)$.

$$\begin{aligned}
&a_1^{(1)} = \{(u_0, v_0) = (\alpha_2 + \alpha_1, 0)\} \in D_0^{(1)}, \quad a_2^{(1)} = \{(u_0, v_0) = (\alpha_2, 0)\} \in D_0^{(1)}, \\
&a_3^{(1)} = \{(u_1, v_1) = (\alpha_3 + \alpha_4, 0)\} \in D_1^{(1)}, \quad a_4^{(1)} = \{(u_1, v_1) = (\alpha_4, 0)\} \in D_1^{(1)}, \\
&a_5^{(1)} = \{(u_\infty, v_\infty) = (\alpha_5, 0)\} \in D_\infty^{(1)}, \quad a_6^{(1)} = \{(u_\infty, v_\infty) = (\alpha_5 + \alpha_6, 0)\} \in D_\infty^{(1)}.
\end{aligned} \tag{15}$$

Let us perform blowing-ups at $a_j^{(1)}$, and denote $D_j^{(2)}$ for the exceptional curves, respectively. We take six coordinate systems (W_j, V_j) around the points at infinity of $D_j^{(2)}$ for $j = 1, 2, 3, 4, 5, 6$, where

$$\begin{aligned}
(W_1, V_1) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2)}{v_0}, v_0\right), \\
(W_2, V_2) &= \left(\frac{u_0 - \alpha_2}{v_0}, v_0\right), \\
(W_3, V_3) &= \left(\frac{u_1 - (\alpha_3 + \alpha_4)}{v_1}, v_1\right), \\
(W_4, V_4) &= \left(\frac{u_1 - \alpha_4}{v_1}, v_1\right), \\
(W_5, V_5) &= \left(\frac{u_\infty - \alpha_5}{v_\infty}, v_\infty\right), \\
(W_6, V_6) &= \left(\frac{u_\infty - (\alpha_5 + \alpha_6)}{v_\infty}, v_\infty\right).
\end{aligned} \tag{16}$$

For the strict transform of $D^{(0)}$, $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-ups, we also denote by same symbol, respectively. Here, the self-intersection number of $D^{(0)}, D_\nu^{(1)}$ and $D_j^{(2)}$ is given by

$$(D^{(0)})^2 = -1. \quad (D_\nu^{(1)})^2 = -3. \tag{17}$$

In order to obtain a minimal compactification of the space of initial conditions, we must blow down along the curve $D^{(0)} \cong \mathbb{P}^1$ to a nonsingular point. For the strict transform of $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-down, we also denote by same symbol, respectively. Let $\tilde{S} \cdots \to \Sigma_2$ be the composition of above nine blowing-ups and one blowing-down. Then, we see that the canonical divisor class $K_{\tilde{S}}$ of $\tilde{S}$ is given by

$$K_{\tilde{S}} := -D_0^{(1)} - D_1^{(1)} - D_\infty^{(1)}, \tag{18}$$

where the self-intersection number of $D_\nu^{(1)} \cong \mathbb{P}^1$ is given by

$$(D_\nu^{(1)})^2 = -2, \tag{19}$$

and

$$D_0^{(1)} \cap D_1^{(1)} \cap D_\infty^{(1)} \neq \varnothing, \quad (D_j^{(1)}, D_k^{(1)}) = 1 \quad (j \neq k). \tag{20}$$

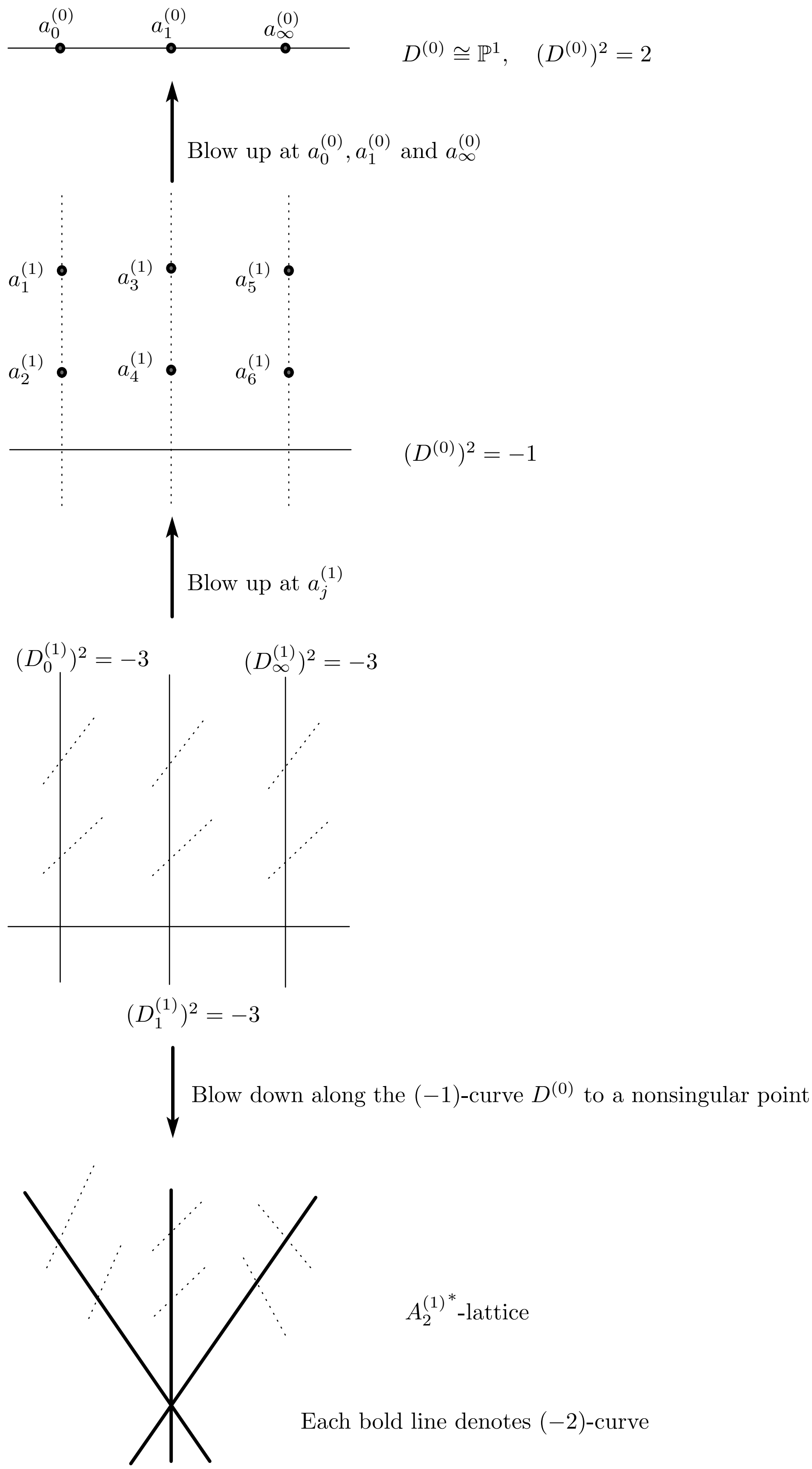


Figure 3: Resolution of accessible singular points

The configuration of the divisor $(-K_{\tilde{S}})_{red}$ on $\tilde{S}$ is of type $A_2^{(1)^*}$ (see Figure 3). And we see that $\tilde{S} - (-K_{\tilde{S}})_{red}$ is covered by eight Zariski open sets

$$\begin{aligned}
&\mathrm{Spec}\ \mathbb{C}[\mathrm{W_j}, \mathrm{V_j}] \quad (\mathrm{j} = 1,2,3,4,5,6),\\
&\mathrm{Spec}\ \mathbb{C}[\mathrm{z_0}, \mathrm{w_0}],\\
&\mathrm{Spec}\ \mathbb{C}[\mathrm{z_1}, \mathrm{w_1}].
\end{aligned} \tag{21}$$

The relations between (W_j, V_j) and (x_j, y_j) are given by

$$(-W_j, V_j) = (x_j, y_j) \quad (j = 1,2,3,4,5,6). \tag{22}$$

We see that the pole divisor of the symplectic 2-form $dp \wedge dq$ coincides with $(-K_{\tilde{S}})_{red}$. Thus, we have completed the proof of Theorems 4.1 and 4.2. □

5 PVI case

Theorem 5.1. *Let us consider a polynomial Hamiltonian system with Hamiltonian $G \in \mathbb{C}(t)[q,p]$. We assume that*
(A1) $deg(G) = 7$ with respect to q, p.
(A2) This system becomes again a polynomial Hamiltonian system in each coordinate rr_i $(i = 0,1,2,3,4)$:

$$\begin{aligned}
&rr_0 : x_0 = q + \frac{\alpha_0 - \alpha_4}{p} + \frac{t}{p^2},\ y_0 = p,\\
&rr_1 : x_1 = -(qp - (\alpha_1 + \alpha_2 + \alpha_4))p,\ y_1 = \frac{1}{p},\\
&rr_2 : x_2 = -(qp - (\alpha_2 + \alpha_4))p,\ y_2 = \frac{1}{p},\\
&rr_3 : x_3 = q + \frac{\alpha_3 - \alpha_4}{p} + \frac{1}{p^2},\ y_3 = p,\\
&rr_4 : x_4 = -(qp - \alpha_4)p,\ y_4 = \frac{1}{p}.
\end{aligned} \tag{23}$$

Then such a system coincides with the system

$$\begin{aligned}
&\frac{dq}{dt} = \frac{\partial G}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial G}{\partial q},\\
&G := -\frac{q^3p^4}{t(t-1)} - \frac{(\alpha_0 + \alpha_3 - 2\alpha_4 - 1)q^2p^3}{t(t-1)}\\
&- \frac{(t+1)q^2p^2}{t(t-1)} - \frac{(\alpha_1\alpha_2 + \alpha_2^2 + 2\alpha_4 - 2\alpha_0\alpha_4 - 2\alpha_3\alpha_4 + \alpha_4^2)qp^2}{t(t-1)}\\
&- \frac{\{(\alpha_3 - \alpha_4)t + \alpha_0 - \alpha_4 - 1\}qp}{t(t-1)} - \frac{q}{t-1} + \frac{\alpha_4(\alpha_2 + \alpha_4)(\alpha_1 + \alpha_2 + \alpha_4)p}{t(t-1)},
\end{aligned} \tag{24}$$

where the constant parameters α_i satisfy the relation:

$$\alpha_0 + \alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 = 1. \tag{25}$$

This Hamiltonian G is equivalent to well-known Hamiltonian H_{VI} of the Painlevé VI system by the birational and symplectic transformation φ

$$\varphi : Q = -(qp - \alpha_4)p, \quad P = \frac{1}{p},$$

where H_{VI} is explicitly given by

$$\begin{aligned}
&H_{VI}(q,p,t;\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)\\
&= \frac{1}{t(t-1)}[p^2(q-t)(q-1)q - \{(\alpha_0 - 1)(q-1)q + \alpha_3(q-t)q\\
&+ \alpha_4(q-t)(q-1)\}p + \alpha_2(\alpha_1 + \alpha_2)(q-t)] \quad (\alpha_0 + \alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 = 1).
\end{aligned} \tag{26}$$

Theorem 5.1 can be checked by a direct calculation.

Theorem 5.2. *The system* (24) *is invariant under the following transformations, whose generators* $w_i,\ i=0,1,2,3,4$, *are given by*

$$\begin{aligned}
&w_0(q,p,t;\alpha_0,\alpha_1,\dots,\alpha_4)\rightarrow\left((1-t)\left(q+\frac{\alpha_0-\alpha_4}{p}+\frac{t}{p^2}\right),\frac{p}{1-t},\frac{t}{t-1};\alpha_4,\alpha_1,\alpha_2,\alpha_3,\alpha_0\right),\\
&w_1(q,p,t;\alpha_0,\alpha_1,\dots,\alpha_4)\rightarrow(q,p,t;\alpha_0,-\alpha_1,\alpha_2+\alpha_1,\alpha_3,\alpha_4),\\
&w_2(q,p,t;\alpha_0,\alpha_1,\dots,\alpha_4)\rightarrow(q,p,t;\alpha_0+\alpha_2,\alpha_1+\alpha_2,-\alpha_2,\alpha_3+\alpha_2,\alpha_4+\alpha_2),\\
&w_3(q,p,t;\alpha_0,\alpha_1,\dots,\alpha_4)\rightarrow\left(-\left(q+\frac{\alpha_3-\alpha_4}{p}+\frac{1}{p^2}\right),-p,1-t;\alpha_0,\alpha_1,\alpha_2,\alpha_4,\alpha_3\right),\\
&w_4(q,p,t;\alpha_0,\alpha_1,\dots,\alpha_4)\rightarrow\left(q,p-\frac{\alpha_4}{q},t;\alpha_0,\alpha_1,\alpha_2+\alpha_4,\alpha_3,-\alpha_4\right).
\end{aligned}\tag{27}$$

Theorem 5.2 can be checked by a direct calculation.

6 Main results of the system with $\tilde{W}(E_7^{(1)})$-symmetry

In this section, by using a relation between holomorphy property and Lax equation, we try to make a second-order polynomial Hamiltonian system with symmetry of the affine Weyl group of type $E_7^{(1)}$. However, for a while, we have not succeeded.

By changing our idea, in the process of construction of the system with $\tilde{W}(E_6^{(1)})$-symmetry, we find the following realtions between symmetry

$$\begin{aligned}
&s_1:(q,p,t;\alpha_1,\alpha_2,\alpha_3)\rightarrow\left(q,p-\frac{\alpha_1}{q},t;-\alpha_1,\alpha_2+\alpha_1,\alpha_3\right),\\
&s_2:(q,p,t;\alpha_1,\alpha_2,\alpha_3)\rightarrow(q,p,t;\alpha_1+\alpha_2,-\alpha_2,\alpha_3+\alpha_2),\\
&s_3:(q,p,t;\alpha_1,\alpha_2,\alpha_3)\rightarrow(q,p,t;\alpha_1,\alpha_2+\alpha_3,-\alpha_3),
\end{aligned}$$

and holomorphy conditions

$$\begin{aligned}
&r_1:x_1=-(pq-\alpha_1)p,\ y_1=1/p,\\
&r_2:x_2=-(pq-(\alpha_1+\alpha_2))p,\ y_2=1/p,\\
&r_3:x_3=-(pq-(\alpha_1+\alpha_2+\alpha_3))p,\ y_3=1/p.
\end{aligned}$$

By using this key property, we try to make a representation of the affine Weyl group symmetry of type $E_7^{(1)}$ and associated holomorphy conditoins.

In this paper, we find and study a 7-parameter family of polynomial Hamiltonian systems of second order. This system admits extended affine Weyl group symmetry of type $E_7^{(1)}$ as the group of its Bäcklund transformations (see Figure 4). This system is the first example which gave second-order polynomial Hamiltonian systems with $\tilde{W}(E_7^{(1)})$-symmetry.

By eliminating p or q, we obtain the second-order ordinary differential equation. However, its form is not normal (cf. [8, 9]).

We also show that after a series of explicit blowing-ups at ten points including the infinitely near points of the Hirzebruch surface Σ_2 (see Figure 5) and two times blowing-downs along the (-1)-curve to a nonsingular point (see Figure 6), respectively, we obtain the rational surface $\tilde{S}$ and a birational morphism

$$\varphi:\tilde{S}=S_{12}\leftarrow S_{11}\leftarrow S_{10}\rightarrow\cdots\rightarrow S_1\rightarrow\Sigma_2.$$

Here, $-K_{\Sigma_2}=2H,\ H\cong\mathbb{P}^1,\ (H)^2=2$. In order to obtain a minimal compactification of the space of initial conditions, we must blow down along the (-1)-curves.

Its canonical divisor $K_{\tilde{S}}$ of $\tilde{S}$ is given by

$$K_{\tilde{S}}=-\sum_{i=1}^{2}E_i,\quad (E_i)^2=-2,\quad E_i\cong\mathbb{P}^1,\quad E_1\cap E_2\neq\varnothing,\quad (E_1,E_2)=1.\tag{28}$$

This configuration of $(-K_{\tilde{S}})_{red}$ is type A_2 (see Figure 4). This type of rational surface $\tilde{S}$ does not appear in the list of Painlevé equations (see [7]).
The space of initial conditions S is obtained by gluing nine copies of $\mathbb{C}^2$

$$\begin{aligned} S &= \tilde{S} - (-K_{\tilde{S}})_{red} \\ &= \mathbb{C}^2 \cup \bigcup_{i=0}^{7} \mathbb{C}^2 \end{aligned} \tag{29}$$

via the birational and symplectic transformations r_j (see Theorem 7.1).
This system is the first example whose minimal model $\tilde{S}$ is the rational surface of type A_2.
The author believes that this system can be obtained by holonomic deformation of the 4rd-order linear ordinary differential equation

$$\frac{d^4y}{dx^4} + a_1(x)\frac{d^3y}{dx^3} + a_2(x)\frac{d^2y}{dx^2} + a_3(x)\frac{dy}{dx} + a_4(x)y = 0 \quad (a_i \in \mathbb{C}(x)) \tag{30}$$

satisfying the Riemann scheme:

$$\begin{pmatrix} x=0 & x=1 & x=q_1 & x=q_2 & x=q_3 & x=\infty \\ 0 & 0 & 0 & 0 & 0 & \alpha_0 \\ \alpha_1 & \alpha_4 & 1 & 1 & 1 & \alpha_0 \\ \alpha_1+\alpha_2 & \alpha_4+\alpha_5 & 2 & 2 & 2 & \alpha_0+\alpha_7 \\ \alpha_1+\alpha_2+\alpha_3 & \alpha_4+\alpha_5+\alpha_6 & 4 & 4 & 4 & \alpha_0+\alpha_7 \end{pmatrix}, \tag{31}$$

where each $x = q_i$ is an apparent singular point.
The author conjectures that three apparent singular points $x = q_i$ satisfy $q_i \in \mathbb{C}(t)(q)$ or $q_i = q$ $(i = 1, 2, 3)$.

7 Holomorphy

Theorem 7.1. *Let us consider a polynomial Hamiltonian system with Hamiltonian $I \in \mathbb{C}(t)[q, p]$. We assume that*
(A1) $deg(I) = 10$ with respect to q, p.
(A2) This system becomes again a polynomial Hamiltonian system in each coordinate r_i $(i = 0, 1, \ldots, 6)$:

$$\begin{aligned} &r_0 : x_0 = 1/q,\ y_0 = -(qp + \alpha_0)q, \\ &r_1 : x_1 = -(pq - \alpha_1)p,\ y_1 = 1/p, \\ &r_2 : x_2 = -(pq - (\alpha_1 + \alpha_2))p,\ y_2 = 1/p, \\ &r_3 : x_3 = -(pq - (\alpha_1 + \alpha_2 + \alpha_3))p,\ y_3 = 1/p, \\ &r_4 : x_4 = -(p(q-1) - \alpha_4)p,\ y_4 = 1/p, \\ &r_5 : x_5 = -(p(q-1) - (\alpha_4 + \alpha_5))p,\ y_5 = 1/p, \\ &r_6 : x_6 = -(p(q-1) - (\alpha_4 + \alpha_5 + \alpha_6))p,\ y_6 = 1/p \end{aligned}$$

(A3) In addition to the assumption (A2), the Hamiltonian system in the coordinate r_0 becomes again a polynomial Hamiltonian system in the coordinate r_7:

$$r_7 : x_7 = -(x_0y_0 - \alpha_7)y_0,\ y_7 = 1/y_0.$$

Then such a system coincides with the system

$$
\begin{aligned}
&\frac{dq}{dt}=\frac{\partial I}{\partial p}, \quad \frac{dp}{dt}=-\frac{\partial I}{\partial q},\\
I :=&(q-1)^3q^3p^4-(q-1)^2q^2\{(3\alpha_1+2\alpha_2+\alpha_3+3\alpha_4+2\alpha_5+\alpha_6)q-3\alpha_1-2\alpha_2-\alpha_3\}p^3\\
&+(q-1)q\{(6\alpha_0^2+6\alpha_0\alpha_7+\alpha_7^2)q^2+(-6\alpha_0^2-3\alpha_1^2-4\alpha_1\alpha_2-\alpha_2^2-2\alpha_1\alpha_3-\alpha_2\alpha_3+3\alpha_4^2+4\alpha_4\alpha_5\\
&+\alpha_5^2+2\alpha_4\alpha_6+\alpha_5\alpha_6-6\alpha_0\alpha_7-\alpha_7^2)q+3\alpha_1^2+\alpha_2^2+\alpha_2\alpha_3+\alpha_1(4\alpha_2+2\alpha_3)\}p^2\\
&fp+\alpha_1(\alpha_1+\alpha_2)(\alpha_1+\alpha_2+\alpha_3)p+\alpha_0^2\{9\alpha_0^2+\alpha_7^2+\alpha_0(6\alpha_1+4\alpha_2+2\alpha_3+6\alpha_4+4\alpha_5+2\alpha_6+6\alpha_7)\}q^2\\
&+\alpha_0[15\alpha_0^3+3\alpha_1^2\alpha_7+\alpha_2^2\alpha_7+3\alpha_4^2\alpha_7+\alpha_5^2\alpha_7+\alpha_6\alpha_7^2+\alpha_7^3\\
&+\alpha_0^2(25\alpha_1+14\alpha_2+3\alpha_3+15\alpha_4+10\alpha_5+5\alpha_6+12\alpha_7)+\alpha_5(\alpha_6\alpha_7+2\alpha_7^2)\\
&+\alpha_2(3\alpha_4\alpha_7+2\alpha_5\alpha_7+\alpha_6\alpha_7+2\alpha_7^2)+\alpha_4(4\alpha_5\alpha_7+2\alpha_6\alpha_7+3\alpha_7^2)\\
&+\alpha_1(3\alpha_2\alpha_7+6\alpha_4\alpha_7+4\alpha_5\alpha_7+2\alpha_6\alpha_7+4\alpha_7^2)+\alpha_0\{9\alpha_1^2+3\alpha_2^2+3\alpha_4^2+\alpha_5^2+3\alpha_6\alpha_7+4\alpha_7^2)\\
&+\alpha_5(\alpha_6+6\alpha_7)+\alpha_2(\alpha_3+6\alpha_4+4\alpha_5+2\alpha_6+8\alpha_7)+\alpha_4(4\alpha_5+2\alpha_6+9\alpha_7)\\
&+\alpha_1(10\alpha_2+2\alpha_3+12\alpha_4+8\alpha_5+4\alpha_6+16\alpha_7)\}]q,
\end{aligned} \tag{32}
$$

where f is explicitly given by

$$
\begin{aligned}
f =&\alpha_0\{16\alpha_0^2+2\alpha_7^2+\alpha_0(9\alpha_1+6\alpha_2+3\alpha_3+9\alpha_4+6\alpha_5+3\alpha_6+12\alpha_7)\}q^3\\
&+[\alpha_0^2(16\alpha_1+8\alpha_2+9\alpha_4+6\alpha_5+3\alpha_6)\alpha_0^2+3\alpha_1^2\alpha_7+\alpha_2^2\alpha_7+3\alpha_4^2\alpha_7+\alpha_5^2\alpha_7+\alpha_6\alpha_7^2+\alpha_7^3\\
&+\alpha_5(\alpha_6\alpha_7+2\alpha_7^2)+\alpha_2(3\alpha_4\alpha_7+2\alpha_5\alpha_7+\alpha_6\alpha_7+2\alpha_7^2)\\
&+\alpha_4(4\alpha_5\alpha_7+2\alpha_6\alpha_7+3\alpha_7^2)+\alpha_1(3\alpha_2\alpha_7+6\alpha_4\alpha_7+4\alpha_5\alpha_7+2\alpha_6\alpha_7+4\alpha_7^2)\\
&+\alpha_0\{6\alpha_1^2+2\alpha_2^2+6\alpha_4^2+2\alpha_5^2+3\alpha_6\alpha_7+2\alpha_7^2+\alpha_5(2\alpha_6+6\alpha_7)+\alpha_2(6\alpha_4+4\alpha_5+2\alpha_6+8\alpha_7)\\
&+\alpha_4(8\alpha_5+4\alpha_6+9\alpha_7)+\alpha_1(6\alpha_2+12\alpha_4+8\alpha_5+4\alpha_6+16\alpha_7)\}]q^2\\
&+[-16\alpha_0^3-\alpha_1^3-\alpha_4^3+\alpha_0^2(-25\alpha_1-14\alpha_2-3\alpha_3-18\alpha_4-12\alpha_5-6\alpha_6-12\alpha_7)\\
&+\alpha_1^2(-2\alpha_2-\alpha_3-3\alpha_7)+\alpha_4^2(-2\alpha_5-\alpha_6-3\alpha_7)-\alpha_2^2\alpha_7-\alpha_5^2\alpha_7-\alpha_6\alpha_7^2-\alpha_7^3\\
&+\alpha_0\{-6\alpha_1^2-2\alpha_2^2-6\alpha_4^2-2\alpha_5^2+\alpha_1(-6\alpha_2-12\alpha_4-8\alpha_5-4\alpha_6-16\alpha_7)+\alpha_4(-8\alpha_5-4\alpha_6-9\alpha_7)\\
&+\alpha_2(-6\alpha_4-4\alpha_5-2\alpha_6-8\alpha_7)+\alpha_5(-2\alpha_6-6\alpha_7)-3\alpha_6\alpha_7-4\alpha_7^2\}\\
&+\alpha_1(-\alpha_2^2+\alpha_2(-\alpha_3-3\alpha_7)-6\alpha_4\alpha_7-4\alpha_5\alpha_7-2\alpha_6\alpha_7-4\alpha_7^2)+\alpha_5(-\alpha_6\alpha_7-2\alpha_7^2)\\
&+\alpha_4(-\alpha_5^2+\alpha_5(-\alpha_6-4\alpha_7)-2\alpha_6\alpha_7-3\alpha_7^2)+\alpha_2(-3\alpha_4\alpha_7-2\alpha_5\alpha_7-\alpha_6\alpha_7-2\alpha_7^2)]q.
\end{aligned} \tag{33}
$$

Here, the constant parameters α_i satisfy the relation:

$$4\alpha_0+3\alpha_1+2\alpha_2+\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+2\alpha_7=0. \tag{34}$$

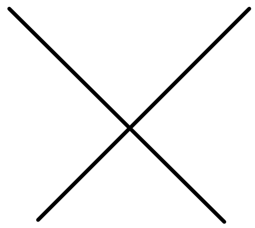
Since each transformation r_i is symplectic, the system (32) is transformed into a Hamiltonian system, whose Hamiltonian may have poles. It is remarkable that the transformed system becomes again a polynomial system for any $i=0,1,\dots,7$.
The holomorphy conditions $(A2),(A3)$ are new. Theorem 7.1 can be checked by a direct calculation.

Proposition 7.1. *The Hamiltonian I is its first integral.*

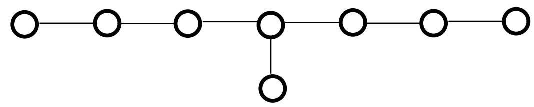
Remark 7.1. *For the Hamiltonian system in each coordinate system (x_i,y_i) $(i=0,1,\dots,7)$ given by $(A2)$ and $(A3)$ in Theorem 7.1, by eliminating x_i or y_i, we obtain the second-order ordinary differential equation. However, its form is not normal* (cf. [8, 9]).

8 Symmetry

Theorem 8.1. *The system* (32) *admits extended affine Weyl group symmetry of type $E_7^{(1)}$ as the group of its Bäcklund transformations whose generators s_i, $i=0,1,\dots,7$ and π are explicitly given as follows*:



A_2-lattice



Dynkin diagram of type $E_7^{(1)}$

Figure 4: A_2-lattice and Dynkin diagram of type $E_7^{(1)}$

with the notation $(*) := (q,p,t;\alpha_0,\alpha_1,\ldots,\alpha_7)$,

$$\begin{aligned}
s_0 &: (*) \to \left(q+\frac{\alpha_0}{p},p,t;-\alpha_0,\alpha_1+\alpha_0,\alpha_2,\alpha_3,\alpha_4+\alpha_0,\alpha_5,\alpha_6,\alpha_7+\alpha_0\right),\\
s_1 &: (*) \to \left(q,p-\frac{\alpha_1}{q},t;\alpha_0+\alpha_1,-\alpha_1,\alpha_2+\alpha_1,\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7\right),\\
s_2 &: (*) \to (q,p,t;\alpha_0,\alpha_1+\alpha_2,-\alpha_2,\alpha_3+\alpha_2,\alpha_4,\alpha_5,\alpha_6,\alpha_7),\\
s_3 &: (*) \to (q,p,t;\alpha_0,\alpha_1,\alpha_2+\alpha_3,-\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7),\\
s_4 &: (*) \to \left(q,p-\frac{\alpha_4}{q-1},t;\alpha_0+\alpha_4,\alpha_1,\alpha_2,\alpha_3,-\alpha_4,\alpha_5+\alpha_4,\alpha_6,\alpha_7\right),\\
s_5 &: (*) \to (q,p,t;\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4+\alpha_5,-\alpha_5,\alpha_6+\alpha_5,\alpha_7),\\
s_6 &: (*) \to (q,p,t;\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5+\alpha_6,-\alpha_6,\alpha_7),\\
s_7 &: (*) \to (q,p,t;\alpha_0+\alpha_7,\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,-\alpha_7),\\
\pi &: (*) \to (1-q,-p,t+1;\alpha_0,\alpha_4,\alpha_5,\alpha_6,\alpha_1,\alpha_2,\alpha_3,\alpha_7),
\end{aligned}$$

where π *is the Dynkin diagram automorphism of type* $E_7^{(1)}$.

The list should be read as

$$\begin{aligned}
&s_0(\alpha_0)=-\alpha_0,\ s_0(\alpha_1)=\alpha_1+\alpha_0,\ s_0(\alpha_2)=\alpha_2,\ s_0(\alpha_3)=\alpha_3,\ s_0(\alpha_4)=\alpha_4+\alpha_0,\\
&s_0(\alpha_5)=\alpha_5,\ s_0(\alpha_6)=\alpha_6,\ s_0(\alpha_7)=\alpha_7+\alpha_0,\\
&s_0(q)=q+\frac{\alpha_0}{p},\ s_0(p)=p,\ s_0(t)=t.
\end{aligned} \tag{35}$$

Theorem 8.1 can be checked by a direct calculation.

9 Space of initial conditions

Theorem 9.1. *After a series of explicit blowing-ups at ten points including the infinitely near points of* Σ_2 *and successive blowing-down along the* (-1)*-curve* ${D^{(0)}}' \cong \mathbb{P}^1$ *and* $D_\infty^{(1)} \cong \mathbb{P}^1$, *we obtain the rational surface* $\tilde{S}$ *of the system* (32) *and a birational morphism* $\varphi : \tilde{S} \cdots \to \Sigma_2$. *Its canonical divisor* $K_{\tilde{S}}$ *of* $\tilde{S}$ *is given by*

$$K_{\tilde{S}} = -D_0^{(1)} - D_1^{(1)}, \quad (D_\nu^{(1)})^2=-2,\ D_\nu^{(1)} \cong \mathbb{P}^1, \tag{36}$$

where the symbol ${D^{(0)}}'$ *denotes the strict transform of* $D^{(0)}$, $D_\nu^{(1)}$ *denote the exceptional divisors and* $-K_{\Sigma_2}=2D^{(0)},\ D^{(0)}\cong\mathbb{P}^1,\ (D^{(0)})^2=2$.

Theorem 9.2. *The space of initial conditions* S *of the system* (32) *is obtained by gluing nine copies of*

$\mathbb{C}^2$*:*

$$\begin{aligned}S &= \tilde{S} - (-K_{\tilde{S}})_{red} \\ &= \mathbb{C}^2 \cup \bigcup_{i=0}^{7} U_j, \\ \mathbb{C}^2 &\ni (q,p), \quad U_j \cong \mathbb{C}^2 \ni (x_j, y_j) \ (j = 0, 1, \ldots, 7)\end{aligned} \tag{37}$$

via the birational and symplectic transformations r_j (see Theorem 8.1).

Proof of Theorems 9.1 and 9.2.
At first, we take the Hirzebruch surface Σ_2. By a direct calculation, we see that the system (32) has three accessible singular points $a_\nu^{(0)} \in D^{(0)} \quad (\nu = 0, 1, \infty)$:

$$\begin{aligned}a_\nu^{(0)} &= \{(z_2, w_2) = (\nu, 0)\} \in U_2 \cap D^{(0)} \ (\nu = 0, 1), \\ a_\infty^{(0)} &= \{(z_3, w_3) = (0, 0)\} \in U_3 \cap D^{(0)}.\end{aligned} \tag{38}$$

We perform blowing-ups in Σ_2 at $a_\nu^{(0)}$, and let $D_\nu^{(1)}$ be the exceptional curves of the blowing-ups at $a_\nu^{(0)}$ for $\nu = 0, 1, \infty$. We can take three coordinate systems (u_ν, v_ν) around the points at infinity of the exceptional curves $D_\nu^{(1)} \quad (\nu = 0, 1, \infty)$, where

$$\begin{aligned}(u_\nu, v_\nu) &= \left(\frac{z_2 - \nu}{w_2}, w_2\right) \ (\nu = 0, 1), \\ (u_\infty, v_\infty) &= \left(\frac{z_3}{w_3}, w_3\right).\end{aligned} \tag{39}$$

Note that $\{(u_\nu, v_\nu) | v_\nu = 0\} \subset D_\nu^{(1)}$ for $\nu = 0, 1, \infty$. By a direct calculation, we see that the system (6) has seven accessible singular points $a_\nu^{(1)}$ for $\nu = 1, 2, 3, 4, 5, 6, 7$ in $D_\nu^{(1)} \cong \mathbb{P}^1$ $(\nu = 0, 1, \infty)$.

$$\begin{aligned}&a_1^{(1)} = \{(u_0, v_0) = (\alpha_1, 0)\} \in D_0^{(1)}, \quad a_2^{(1)} = \{(u_0, v_0) = (\alpha_1 + \alpha_2, 0)\} \in D_0^{(1)}, \\ &a_3^{(1)} = \{(u_0, v_0) = (\alpha_1 + \alpha_2 + \alpha_3, 0)\} \in D_0^{(1)}, \quad a_4^{(1)} = \{(u_1, v_1) = (\alpha_4, 0)\} \in D_1^{(1)}, \\ &a_5^{(1)} = \{(u_1, v_1) = (\alpha_4 + \alpha_5, 0)\} \in D_1^{(1)}, \quad a_6^{(1)} = \{(u_1, v_1) = (\alpha_4 + \alpha_5 + \alpha_6, 0)\} \in D_1^{(1)}, \\ &a_7^{(1)} = \{(u_\infty, v_\infty) = (\alpha_7, 0)\} \in D_\infty^{(1)}.\end{aligned} \tag{40}$$

Let us perform blowing-ups at $a_j^{(1)}$, and denote $D_j^{(2)}$ for the exceptional curves, respectively. We take seven coordinate systems (W_j, V_j) around the points at infinity of $D_j^{(2)}$ for $j = 1, 2, 3, 4, 5, 6, 7$, where

$$\begin{aligned}(W_1, V_1) &= \left(\frac{u_0 - \alpha_1}{v_0}, v_0\right), \\ (W_2, V_2) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2)}{v_0}, v_0\right), \\ (W_3, V_3) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2 + \alpha_3)}{v_0}, v_0\right), \\ (W_4, V_4) &= \left(\frac{u_1 - \alpha_4}{v_1}, v_1\right), \\ (W_5, V_5) &= \left(\frac{u_1 - (\alpha_4 + \alpha_5)}{v_1}, v_1\right), \\ (W_6, V_6) &= \left(\frac{u_1 - (\alpha_4 + \alpha_5 + \alpha_6)}{v_1}, v_1\right), \\ (W_7, V_7) &= \left(\frac{u_\infty - \alpha_7}{v_\infty}, v_\infty\right).\end{aligned} \tag{41}$$

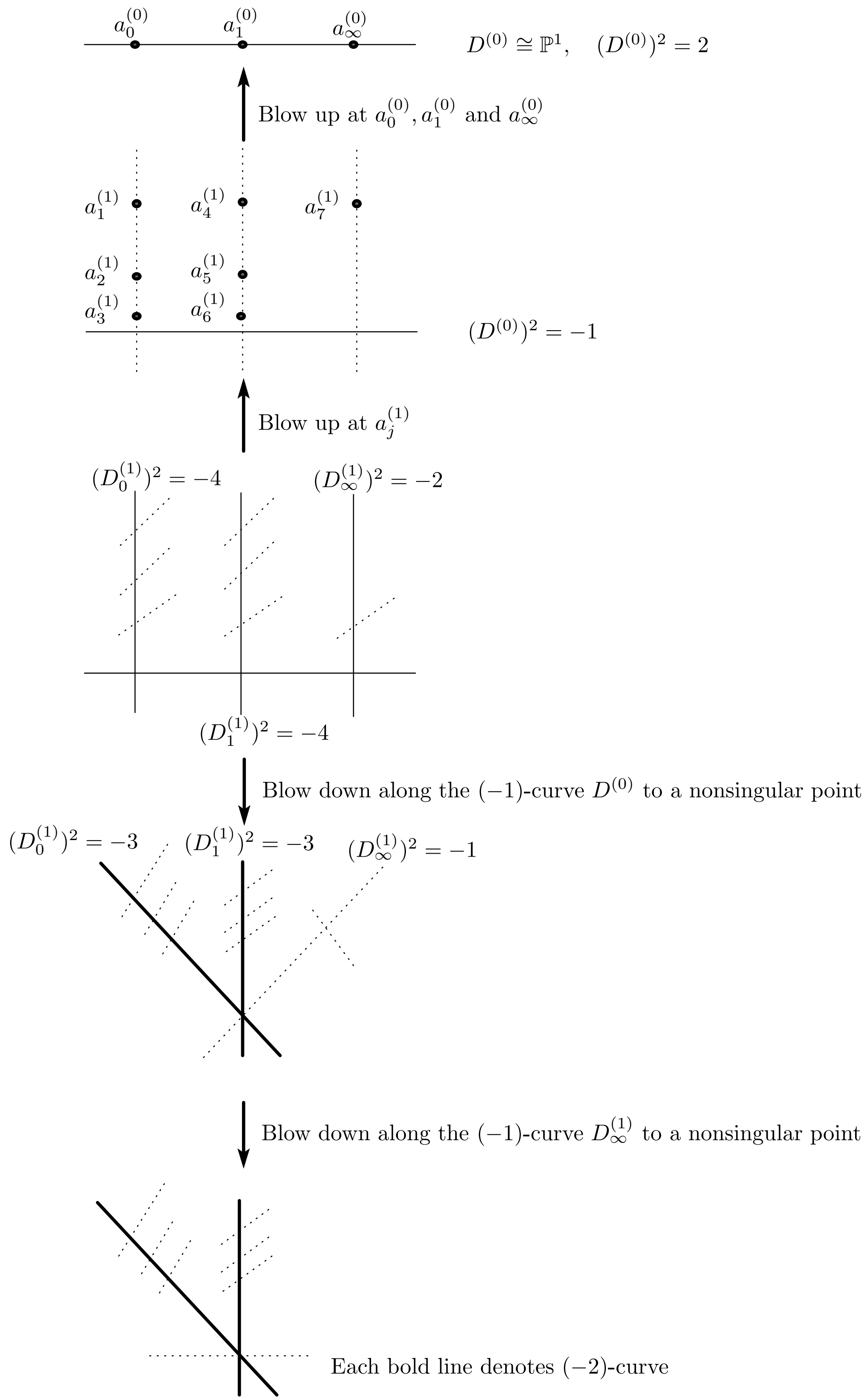


Figure 5: Resolution of accessible singular points

For the strict transform of $D^{(0)}$, $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-ups, we also denote by same symbol, respectively. Here, the self-intersection number of $D^{(0)}, D_\nu^{(1)}$ is given by

$$(D^{(0)})^2 = -1. \quad (D_0^{(1)})^2 = (D_1^{(1)})^2 = -4, \quad (D_\infty^{(1)})^2 = -2. \tag{42}$$

In order to obtain a minimal compactification of the space of initial conditions, we must blow down along the (-1)-curve $D^{(0)} \cong \mathbb{P}^1$ to a nonsingular point. For the strict transform of $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-down, we also denote by same symbol, respectively. Here, the self-intersection number of $D_\nu^{(1)}$ is given by

$$(D_0^{(1)})^2 = (D_1^{(1)})^2 = -3, \quad (D_\infty^{(1)})^2 = -1. \tag{43}$$

We must blow down again along the (-1)-curve $D_\infty^{(1)} \cong \mathbb{P}^1$ to a nonsingular point. For the strict transform of $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-down, we also denote by same symbol, respectively. Here, the self-intersection number of $D_\nu^{(1)}$ is given by

$$(D_0^{(1)})^2 = (D_1^{(1)})^2 = -2. \tag{44}$$

Let $\tilde{S} \cdots \to \Sigma_2$ be the composition of above ten times blowing-ups and two times blowing-downs. Then, we see that the canonical divisor class $K_{\tilde{S}}$ of $\tilde{S}$ is given by

$$K_{\tilde{S}} := -D_0^{(1)} - D_1^{(1)}, \tag{45}$$

where the self-intersection number of $D_\nu^{(1)} \cong \mathbb{P}^1$ is given by

$$(D_\nu^{(1)})^2 = -2, \tag{46}$$

and

$$D_0^{(1)} \cap D_1^{(1)} \neq \varnothing, \quad (D_0^{(1)}, D_1^{(1)}) = 1. \tag{47}$$

The configuration of the divisor $(-K_{\tilde{S}})_{red}$ on $\tilde{S}$ is of type A_2. And we see that $\tilde{S} - (-K_{\tilde{S}})_{red}$ is covered by nine Zariski open sets

$$\begin{aligned} &\mathrm{Spec}\ \mathbb{C}[\mathrm{W_j}, \mathrm{V_j}] \quad (\mathrm{j} = 1,2,3,4,5,6,7),\\ &\mathrm{Spec}\ \mathbb{C}[\mathrm{z_0}, \mathrm{w_0}],\\ &\mathrm{Spec}\ \mathbb{C}[\mathrm{z_1}, \mathrm{w_1}]. \end{aligned} \tag{48}$$

The relations between (W_j, V_j) and (x_j, y_j) are given by

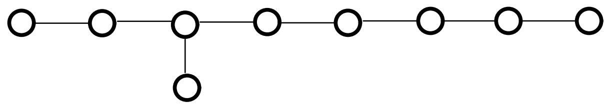
$$(-W_j, V_j) = (x_j, y_j) \quad (j = 1,2,3,4,5,6,7). \tag{49}$$

We see that the pole divisor of the symplectic 2-form $dp \wedge dq$ coincides with $(-K_{\tilde{S}})_{red}$. Thus, we have completed the proof of Theorems 9.1 and 9.2. □

10 Main results of the system with $W(E_8^{(1)})$-symmetry

By using the key property, we try to make a second-order polynomial Hamiltonian system with symmetry of the affine Weyl group of type $E_8^{(1)}$.
At first, we make a representation of affine Weyl group of type $E_8^{(1)}$. Next, we make holomorphy conditions r_i $(i = 0,1,\ldots,8)$ associated with it.

Problem 10.1. *Can we make a polynomial Hamiltonian system with Hamiltonian $I \in \mathbb{C}(t)[q,p]$ satisfying the following condition (A)?:*
(A):This system becomes again a polynomial Hamiltonian system in each coordinate r_i $(i = 0,1,\ldots,8)$



Dynkin diagram of type $E_8^{(1)}$

Figure 6:

Before we solve this problem, we construct the space of initial conditions characterized by holomorphy conditions r_i $(i = 0, 1, \dots, 8)$. After a series of explicit blowing-ups at eleven points including the infinitely near points of the Hirzebruch surface Σ_2 (see Figure 2) and three times blowing-downs along the (-1)-curve to a nonsingular point (see Figure 7), respectively, we obtain the smooth rational surface $\tilde{S}$ and a birational morphism

$$\varphi : \tilde{S} = S_{14} \leftarrow S_{13} \leftarrow S_{12} \leftarrow S_{11} \rightarrow \cdots \rightarrow S_1 \rightarrow \Sigma_2.$$

Here, $-K_{\Sigma_2} = 2H,\ H \cong \mathbb{P}^1,\ (H)^2 = 2$. In order to obtain a minimal compactification of the space of initial conditions, we must blow down along the (-1)-curves. Its canonical divisor $K_{\tilde{S}}$ of $\tilde{S}$ is given by

$$K_{\tilde{S}} = -E, \quad E \cong \mathbb{P}^1, \quad (E)^2 = -3. \tag{50}$$

In the case of Painlevé equations, each component of the anti-canonical divisor $-K_{\tilde{S}}$ is (-2)-curve. However, in this case, it is (-3)-curve. In this vein, we take a poor view of holomorphy conditions r_i $(i = 0, 1, \dots, 8)$.

However, we can obtain a 8-parameter family of polynomial Hamiltonian systems determined by r_i $(i = 0, 1, \dots, 8)$. Suprisingly, this system admits the affine Weyl group symmetry of type $E_8^{(1)}$ as the group of its Bäcklund transformations (see Figure 6). This system is the first example which gave second-order polynomial Hamiltonian systems with $W(E_8^{(1)})$-symmetry.

By eliminating p or q, we obtain the second-order ordinary differential equation. However, its form is not normal (cf. [8, 9]).

The space of initial conditions S is obtained by gluing ten copies of $\mathbb{C}^2$

$$\begin{aligned} S &= \tilde{S} - (-K_{\tilde{S}})_{red} \\ &= \mathbb{C}^2 \cup \bigcup_{i=0}^{8} \mathbb{C}^2 \end{aligned} \tag{51}$$

via the birational and symplectic transformations r_j (see Theorem 11.1).

The author believes that this system can be obtained by holonomic deformation of the 6rd-order linear ordinary differential equation

$$\frac{d^6y}{dx^6} + a_1(x)\frac{d^5y}{dx^5} + a_2(x)\frac{d^4y}{dx^4} + a_3(x)\frac{d^3y}{dx^3} + a_4(x)\frac{d^2y}{dx^2} + a_5(x)\frac{dy}{dx} + a_6(x)y = 0 \quad (a_i \in \mathbb{C}(x)) \tag{52}$$

satisfying the Riemann scheme:

$$\begin{pmatrix} x=0 & x=1 & x=q_i & x=\infty \\ 0 & 0 & 0 & \alpha_0 \\ \alpha_1 & 0 & 1 & \alpha_0 \\ \alpha_1+\alpha_2 & \alpha_6 & 2 & \alpha_0 \\ \alpha_1+\alpha_2+\alpha_3 & \alpha_6 & 3 & \alpha_0+\alpha_8 \\ \alpha_1+\alpha_2+\alpha_3+\alpha_4 & \alpha_6+\alpha_7 & 4 & \alpha_0+\alpha_8 \\ \alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5 & \alpha_6+\alpha_7 & 6 & \alpha_0+\alpha_8 \end{pmatrix}, \tag{53}$$

where each $x = q_i$ $(i = 1, 2, \dots, 10)$ is an apparent singular point.

The author conjectures that ten apparent singular points $x = q_i$ satisfy $q_i \in \mathbb{C}(t)(q)$ or $q_i = q$.

11 Holomorphy

Theorem 11.1. *Let us consider a polynomial Hamiltonian system with Hamiltonian $I \in \mathbb{C}(t)[q,p]$. We assume that*
$(C1)$ $deg(I) = 15$ with respect to q, p.
$(C2)$ This system becomes again a polynomial Hamiltonian system in each coordinate r_i $(i = 0, 1, \ldots, 7)$:

$$
\begin{aligned}
&r_0 : x_0 = 1/q,\ \ y_0 = -(qp + \alpha_0)q,\\
&r_1 : x_1 = -(pq - \alpha_1)p,\ \ y_1 = 1/p,\\
&r_2 : x_2 = -(pq - (\alpha_1 + \alpha_2))p,\ \ y_2 = 1/p,\\
&r_3 : x_3 = -(pq - (\alpha_1 + \alpha_2 + \alpha_3))p,\ \ y_3 = 1/p,\\
&r_4 : x_4 = -(pq - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4))p,\ \ y_4 = 1/p,\\
&r_5 : x_5 = -(pq - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5))p,\ \ y_5 = 1/p,\\
&r_6 : x_6 = -(p(q-1) - \alpha_6)p,\ \ y_6 = 1/p,\\
&r_7 : x_7 = -(p(q-1) - (\alpha_6 + \alpha_7))p,\ \ y_7 = 1/p.
\end{aligned}
$$

$(C3)$ In addition to the assumption $(C2)$, the Hamiltonian system in the coordinate r_0 becomes again a polynomial Hamiltonian system in the coordinate r_7:

$$r_8 : x_8 = -(x_0 y_0 - \alpha_8)y_0,\ \ y_8 = 1/y_0.$$

Then such a system coincides with the system

$$\frac{dq}{dt} = \frac{\partial I}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial I}{\partial q}, \tag{54}$$

$I := p^6(-1+q)^4q^5 + p^5(-1+q)^3q^4(-6\alpha_0 - 4\alpha_6 - 2\alpha_7 - 3\alpha_8 + 3q(2\alpha_0 + \alpha_8)) + p^4(-1+q)^2q^3(-24\alpha_0\alpha_1 - 10\alpha_1^2 - 18\alpha_0\alpha_2 - 15\alpha_1\alpha_2 - 6\alpha_2^2 - 12\alpha_0\alpha_3 - 10\alpha_1\alpha_3 - 8\alpha_2\alpha_3 - 3\alpha_3^2 - 6\alpha_0\alpha_4 - 5\alpha_1\alpha_4 - 4\alpha_2\alpha_4 - 3\alpha_3\alpha_4 - \alpha_4^2 - 16\alpha_1\alpha_6 - 12\alpha_2\alpha_6 - 8\alpha_3\alpha_6 - 4\alpha_4\alpha_6 - 8\alpha_1\alpha_7 - 6\alpha_2\alpha_7 - 4\alpha_3\alpha_7 - 2\alpha_4\alpha_7 - 12\alpha_1\alpha_8 - 9\alpha_2\alpha_8 - 6\alpha_3\alpha_8 - 3\alpha_4\alpha_8 + q(-15\alpha_0^2 + 24\alpha_0\alpha_1 + 10\alpha_1^2 + 18\alpha_0\alpha_2 + 15\alpha_1\alpha_2 + 6\alpha_2^2 + 12\alpha_0\alpha_3 + 10\alpha_1\alpha_3 + 8\alpha_2\alpha_3 + 3\alpha_3^2 + 6\alpha_0\alpha_4 + 5\alpha_1\alpha_4 + 4\alpha_2\alpha_4 + 3\alpha_3\alpha_4 + \alpha_4^2 + 16\alpha_1\alpha_6 + 12\alpha_2\alpha_6 + 8\alpha_3\alpha_6 + 4\alpha_4\alpha_6 + 6\alpha_6^2 + 8\alpha_1\alpha_7 + 6\alpha_2\alpha_7 + 4\alpha_3\alpha_7 + 2\alpha_4\alpha_7 + 6\alpha_6\alpha_7 + \alpha_7^2 - 15\alpha_0\alpha_8 + 12\alpha_1\alpha_8 + 9\alpha_2\alpha_8 + 6\alpha_3\alpha_8 + 3\alpha_4\alpha_8 - 3\alpha_8^2) + 3q^2(5\alpha_0^2 + 5\alpha_0\alpha_8 + \alpha_8^2)) + q\alpha_0(\alpha_0 + \alpha_8)(-44\alpha_0^4 - 120\alpha_0^3\alpha_1 - 122\alpha_0^2\alpha_1^2 - 64\alpha_0\alpha_1^3 - 15\alpha_1^4 - 90\alpha_0^3\alpha_2 - 183\alpha_0^2\alpha_1\alpha_2 - 144\alpha_0\alpha_1^2\alpha_2 - 45\alpha_1^3\alpha_2 - 66\alpha_0^2\alpha_2^2 - 102\alpha_0\alpha_1\alpha_2^2 - 48\alpha_1^2\alpha_2^2 - 22\alpha_0\alpha_2^3 - 21\alpha_1\alpha_2^3 - 3\alpha_2^4 - 60\alpha_0^3\alpha_3 - 122\alpha_0^2\alpha_1\alpha_3 - 96\alpha_0\alpha_1^2\alpha_3 - 30\alpha_1^3\alpha_3 - 88\alpha_0^2\alpha_2\alpha_3 - 136\alpha_0\alpha_1\alpha_2\alpha_3 - 64\alpha_1^2\alpha_2\alpha_3 - 44\alpha_0\alpha_2^2\alpha_3 - 42\alpha_1\alpha_2^2\alpha_3 - 8\alpha_2^3\alpha_3 - 27\alpha_0^2\alpha_3^2 - 40\alpha_0\alpha_1\alpha_3^2 - 19\alpha_1^2\alpha_3^2 - 26\alpha_0\alpha_2\alpha_3^2 - 25\alpha_1\alpha_2\alpha_3^2 - 7\alpha_2^2\alpha_3^2 - 4\alpha_0\alpha_3^3 - 4\alpha_1\alpha_3^3 - 2\alpha_2\alpha_3^3 - 30\alpha_0^3\alpha_4 - 61\alpha_0^2\alpha_1\alpha_4 - 48\alpha_0\alpha_1^2\alpha_4 - 15\alpha_1^3\alpha_4 - 44\alpha_0^2\alpha_2\alpha_4 - 68\alpha_0\alpha_1\alpha_2\alpha_4 - 32\alpha_1^2\alpha_2\alpha_4 - 22\alpha_0\alpha_2^2\alpha_4 - 21\alpha_1\alpha_2^2\alpha_4 - 4\alpha_2^3\alpha_4 - 27\alpha_0^2\alpha_3\alpha_4 - 40\alpha_0\alpha_1\alpha_3\alpha_4 - 19\alpha_1^2\alpha_3\alpha_4 - 26\alpha_0\alpha_2\alpha_3\alpha_4 - 25\alpha_1\alpha_2\alpha_3\alpha_4 - 7\alpha_2^2\alpha_3\alpha_4 - 6\alpha_0\alpha_3^2\alpha_4 - 6\alpha_1\alpha_3^2\alpha_4 - 3\alpha_2\alpha_3^2\alpha_4 - 5\alpha_0^2\alpha_4^2 - 6\alpha_0\alpha_1\alpha_4^2 - 3\alpha_1^2\alpha_4^2 - 4\alpha_0\alpha_2\alpha_4^2 - 4\alpha_1\alpha_2\alpha_4^2 - \alpha_2^2\alpha_4^2 - 2\alpha_0\alpha_3\alpha_4^2 - 2\alpha_1\alpha_3\alpha_4^2 - \alpha_2\alpha_3\ \alpha_4^2 - 128\alpha_0^3\alpha_6 - 272\alpha_0^2\alpha_1\alpha_6 - 200\alpha_0\alpha_1^2\alpha_6 - 56\alpha_1^3\alpha_6 - 204\alpha_0^2\alpha_2\alpha_6 - 300\alpha_0\alpha_1\alpha_2\alpha_6 - 126\alpha_1^2\alpha_2\alpha_6 - 108\alpha_0\alpha_2^2\alpha_6 - 90\alpha_1\alpha_2^2\alpha_6 - 20\alpha_2^3\alpha_6 - 136\alpha_0^2\alpha_3\alpha_6 - 200\alpha_0\alpha_1\alpha_3\alpha_6 - 84\alpha_1^2\alpha_3\alpha_6 - 144\alpha_0\alpha_2\alpha_3\alpha_6 - 120\alpha_1\alpha_2\alpha_3\alpha_6 - 40\alpha_2^2\alpha_3\alpha_6 - 44\alpha_0\alpha_3^2\alpha_6 - 36\alpha_1\alpha_3^2\alpha_6 - 24\alpha_2\alpha_3^2\alpha_6 - 4\alpha_3^3\alpha_6 - 68\alpha_0^2\alpha_4\alpha_6 - 100\alpha_0\alpha_1\alpha_4\alpha_6 - 42\alpha_1^2\alpha_4\alpha_6 - 72\alpha_0\alpha_2\alpha_4\alpha_6 - 60\alpha_1\alpha_2\alpha_4\alpha_6 - 20\alpha_2^2\alpha_4\alpha_6 - 44\alpha_0\alpha_3\alpha_4\alpha_6 - 36\alpha_1\alpha_3\alpha_4\alpha_6 - 24\alpha_2\alpha_3\alpha_4\alpha_6 - 6\alpha_3^2\alpha_4\alpha_6 - 8\alpha_0\alpha_4^2\alpha_6 - 6\alpha_1\alpha_4^2\alpha_6 - 4\alpha_2\alpha_4^2\alpha_6 - 2\alpha_3\alpha_4^2\alpha_6 - 137\alpha_0^2\alpha_6^2 - 200\alpha_0\alpha_1\alpha_6^2 - 78\alpha_1^2\alpha_6^2 - 150\alpha_0\alpha_2\alpha_6^2 - 117\alpha_1\alpha_2\alpha_6^2 - 42\alpha_2^2\alpha_6^2 - 100\alpha_0\alpha_3\alpha_6^2 - 78\alpha_1\alpha_3\alpha_6^2 - 56\alpha_2\alpha_3\alpha_6^2 - 17\alpha_3^2\alpha_6^2 - 50\alpha_0\alpha_4\alpha_6^2 - 39\alpha_1\alpha_4\alpha_6^2 - 28\alpha_2\alpha_4\alpha_6^2 - 17\alpha_3\alpha_4\alpha_6^2 - 3\alpha_4^2\alpha_6^2 - 64\alpha_0\alpha_6^3 - 48\alpha_1\alpha_6^3 - 36\alpha_2\alpha_6^3 - 24\alpha_3\alpha_6^3 - 12\alpha_4\alpha_6^3 - 11\alpha_6^4 - 64\alpha_0^3\alpha_7 - 136\alpha_0^2\alpha_1\alpha_7 - 100\alpha_0\alpha_1^2\alpha_7 - 28\alpha_1^3\alpha_7 - 102\alpha_0^2\alpha_2\alpha_7 - 150\alpha_0\alpha_1\alpha_2\alpha_7 - 63\alpha_1^2\alpha_2\alpha_7 - 54\alpha_0\alpha_2^2\alpha_7 - 45\alpha_1\alpha_2^2\alpha_7 - 10\alpha_2^3\alpha_7 - 68\alpha_0^2\alpha_3\alpha_7 - 100\alpha_0\alpha_1\alpha_3\alpha_7 - 42\alpha_1^2\alpha_3\alpha_7 - 72\alpha_0\alpha_2\alpha_3\alpha_7 - 60\alpha_1\alpha_2\alpha_3\alpha_7 - 20\alpha_2^2\alpha_3\alpha_7 - 22\alpha_0\alpha_3^2\alpha_7 - 18\alpha_1\alpha_3^2\alpha_7 - 12\alpha_2\alpha_3^2\alpha_7 - 2\alpha_3^3\alpha_7 - 34\alpha_0^2\alpha_4\alpha_7 - 50\alpha_0\alpha_1\alpha_4\alpha_7 - 21\alpha_1^2\alpha_4\alpha_7 - 36\alpha_0\alpha_2\alpha_4\alpha_7 - 30\alpha_1\alpha_2\alpha_4\alpha_7 - 10\alpha_2^2\alpha_4\alpha_7 - 22\alpha_0\alpha_3\alpha_4\alpha_7 - 18\alpha_1\alpha_3\alpha_4\alpha_7 - 12\alpha_2\alpha_3\alpha_4\alpha_7 - 3\alpha_3^2\alpha_4\alpha_7 - 4\alpha_0\alpha_4^2\alpha_7 - 3\alpha_1\alpha_4^2\alpha_7 - 2\alpha_2\alpha_4^2\alpha_7 - \alpha_3\alpha_4^2\ \alpha_7 - 137\alpha_0^2\alpha_6\alpha_7 - 200\alpha_0\alpha_1\alpha_6\alpha_7 - 78\alpha_1^2\alpha_6\alpha_7 - 150\alpha_0\alpha_2\alpha_6\alpha_7 - 117\alpha_1\alpha_2\alpha_6\alpha_7 - 42\alpha_2^2\alpha_6\alpha_7 - 100\alpha_0\alpha_3\alpha_6\alpha_7 - 78\alpha_1\alpha_3\alpha_6\alpha_7 - 56\alpha_2\alpha_3\alpha_6\alpha_7 - 17\alpha_3^2\alpha_6\alpha_7 - 50\alpha_0\alpha_4\alpha_6\alpha_7 - 39\alpha_1\alpha_4\alpha_6\alpha_7 - 28\alpha_2\alpha_4\alpha_6\alpha_7 - 17\alpha_3\alpha_4\alpha_6\alpha_7 - 3\alpha_4^2\alpha_6\alpha_7 - 96\alpha_0\alpha_6^2\alpha_7 - 72\alpha_1\alpha_6^2\alpha_7 - 54\alpha_2\alpha_6^2\alpha_7 - 36\alpha_3\alpha_6^2\alpha_7 - 18\alpha_4\alpha_6^2\alpha_7 - 22\alpha_6^3\alpha_7 - 37\alpha_0^2\alpha_7^2 - 56\alpha_0\alpha_1\alpha_7^2 - 22\alpha_1^2\alpha_7^2 - 42\alpha_0\alpha_2\alpha_7^2 - 33\alpha_1\alpha_2\alpha_7^2 - 12\alpha_2^2\alpha_7^2 - 28\alpha_0\alpha_3\alpha_7^2 - 22\alpha_1\alpha_3\alpha_7^2 - 16\alpha_2\alpha_3\alpha_7^2 - 5\alpha_3^2\alpha_7^2 - 14\alpha_0\alpha_4\alpha_7^2 - 11\alpha_1\alpha_4\alpha_7^2 - 8\alpha_2\alpha_4\alpha_7^2 - 5\alpha_3\alpha_4\alpha_7^2 - \alpha_4^2\alpha_7^2 - 52\alpha_0\alpha_6\alpha_7^2 -$

$40\alpha_1\alpha_6\alpha_7^2-30\alpha_2\alpha_6\alpha_7^2-20\alpha_3\alpha_6\alpha_7^2-10\alpha_4\alpha_6\alpha_7^2-18\alpha_6^2\alpha_7^2-10\alpha_0\alpha_7^3-8\alpha_1\alpha_7^3-6\alpha_2\alpha_7^3-4\alpha_3\alpha_7^3-2\alpha_4\alpha_7^3-7\alpha_6\alpha_7^3-$
$\alpha_7^4-88\alpha_0^3\alpha_8-180\alpha_0^2\alpha_1\alpha_8-122\alpha_0\alpha_1^2\alpha_8-32\alpha_1^3\alpha_8-135\alpha_0^2\alpha_2\alpha_8-183\alpha_0\alpha_1\alpha_2\alpha_8-72\alpha_1^2\alpha_2\alpha_8-66\alpha_0\alpha_2^2\alpha_8-$
$51\alpha_1\alpha_2^2\alpha_8-11\alpha_2^3\alpha_8-90\alpha_0^2\alpha_3\alpha_8-122\alpha_0\alpha_1\alpha_3\alpha_8-48\alpha_1^2\alpha_3\alpha_8-88\alpha_0\alpha_2\alpha_3\alpha_8-68\alpha_1\alpha_2\alpha_3\alpha_8-22\alpha_2^2\alpha_3\alpha_8-$
$27\alpha_0\alpha_3^2\alpha_8-20\alpha_1\alpha_3^2\alpha_8-13\alpha_2\alpha_3^2\alpha_8-2\alpha_3^3\alpha_8-45\alpha_0^2\alpha_4\alpha_8-61\alpha_0\alpha_1\alpha_4\alpha_8-24\alpha_1^2\alpha_4\alpha_8-44\alpha_0\alpha_2\alpha_4\alpha_8-$
$34\alpha_1\alpha_2\alpha_4\alpha_8-11\alpha_2^2\alpha_4\alpha_8-27\alpha_0\alpha_3\alpha_4\alpha_8-20\alpha_1\alpha_3\alpha_4\alpha_8-13\alpha_2\alpha_3\alpha_4\alpha_8-3\alpha_3^2\alpha_4\alpha_8-5\alpha_0\alpha_4^2\alpha_8-3\alpha_1\alpha_4^2\alpha_8-$
$2\alpha_2\alpha_4^2\alpha_8-\alpha_3\alpha_4^2\alpha_8-192\alpha_0^2\alpha_6\alpha_8-272\alpha_0\alpha_1\alpha_6\alpha_8-100\alpha_1^2\alpha_6\alpha_8-204\alpha_0\alpha_2\alpha_6\alpha_8-150\alpha_1\alpha_2\alpha_6\alpha_8-54\alpha_2^2\alpha_6\alpha_8-$
$136\alpha_0\alpha_3\alpha_6\alpha_8-100\alpha_1\alpha_3\alpha_6\alpha_8-72\alpha_2\alpha_3\alpha_6\alpha_8-22\alpha_3^2\alpha_6\alpha_8-68\alpha_0\alpha_4\alpha_6\alpha_8-50\alpha_1\alpha_4\alpha_6\alpha_8-36\alpha_2\alpha_4\alpha_6\alpha_8-$
$22\alpha_3\alpha_4\alpha_6\alpha_8-4\alpha_4^2\alpha_6\alpha_8-137\alpha_0\alpha_6^2\alpha_8-100\alpha_1\alpha_6^2\alpha_8-75\alpha_2\alpha_6^2\alpha_8-50\alpha_3\alpha_6^2\alpha_8-25\alpha_4\alpha_6^2\alpha_8-32\alpha_6^3\alpha_8-$
$96\alpha_0^2\alpha_7\alpha_8-136\alpha_0\alpha_1\alpha_7\alpha_8-50\alpha_1^2\alpha_7\alpha_8-102\alpha_0\alpha_2\alpha_7\alpha_8-75\alpha_1\alpha_2\alpha_7\alpha_8-27\alpha_2^2\alpha_7\alpha_8-68\alpha_0\alpha_3\alpha_7\alpha_8-50\alpha_1\alpha_3\alpha_7\alpha_8-$
$36\alpha_2\alpha_3\alpha_7\alpha_8-11\alpha_3^2\alpha_7\alpha_8-34\alpha_0\alpha_4\alpha_7\alpha_8-25\alpha_1\alpha_4\alpha_7\alpha_8-18\alpha_2\alpha_4\alpha_7\alpha_8-11\alpha_3\alpha_4\alpha_7\alpha_8-2\alpha_4^2\alpha_7\alpha_8-137\alpha_0\alpha_6\alpha_7\alpha_8-$
$100\alpha_1\alpha_6\alpha_7\alpha_8-75\alpha_2\alpha_6\alpha_7\alpha_8-50\alpha_3\alpha_6\alpha_7\alpha_8-25\alpha_4\alpha_6\alpha_7\alpha_8-48\alpha_6^2\alpha_7\alpha_8-37\alpha_0\alpha_7^2\alpha_8-28\alpha_1\alpha_7^2\alpha_8-21\alpha_2\alpha_7^2\alpha_8-$
$14\alpha_3\alpha_7^2\alpha_8-7\alpha_4\alpha_7^2\alpha_8-26\alpha_6\alpha_7^2\alpha_8-5\alpha_7^3\alpha_8-63\alpha_0^2\alpha_8^2-84\alpha_0\alpha_1\alpha_8^2-28\alpha_1^2\alpha_8^2-63\alpha_0\alpha_2\alpha_8^2-42\alpha_1\alpha_2\alpha_8^2-15\alpha_2^2\alpha_8^2-$
$42\alpha_0\alpha_3\alpha_8^2-28\alpha_1\alpha_3\alpha_8^2-20\alpha_2\alpha_3\alpha_8^2-6\alpha_3^2\alpha_8^2-21\alpha_0\alpha_4\alpha_8^2-14\alpha_1\alpha_4\alpha_8^2-10\alpha_2\alpha_4\alpha_8^2-6\alpha_3\alpha_4\alpha_8^2-\alpha_4^2\alpha_8^2-$
$92\alpha_0\alpha_6\alpha_8^2-64\alpha_1\alpha_6\alpha_8^2-48\alpha_2\alpha_6\alpha_8^2-32\alpha_3\alpha_6\alpha_8^2-16\alpha_4\alpha_6\alpha_8^2-33\alpha_6^2\alpha_8^2-46\alpha_0\alpha_7\alpha_8^2-32\alpha_1\alpha_7\alpha_8^2-24\alpha_2\alpha_7\alpha_8^2-$
$16\alpha_3\alpha_7\alpha_8^2-8\alpha_4\alpha_7\alpha_8^2-33\alpha_6\alpha_7\alpha_8^2-9\alpha_7^2\alpha_8^2-19\alpha_0\alpha_8^3-12\alpha_1\alpha_8^3-9\alpha_2\alpha_8^3-6\alpha_3\alpha_8^3-3\alpha_4\alpha_8^3-14\alpha_6\alpha_8^3-7\alpha_7\alpha_8^3-$
$2\alpha_8^4+q^2\alpha_0^2(\alpha_0+\alpha_8)^2+q\alpha_0(\alpha_0+\alpha_8)(11\alpha_0^2+24\alpha_0\alpha_1+10\alpha_1^2+18\alpha_0\alpha_2+15\alpha_1\alpha_2+6\alpha_2^2+12\alpha_0\alpha_3+10\alpha_1\alpha_3+$
$8\alpha_2\alpha_3+3\alpha_3^2+6\alpha_0\alpha_4+5\alpha_1\alpha_4+4\alpha_2\alpha_4+3\alpha_3\alpha_4+\alpha_4^2+16\alpha_0\alpha_6+16\alpha_1\alpha_6+12\alpha_2\alpha_6+8\alpha_3\alpha_6+4\alpha_4\alpha_6+6\alpha_6^2+$
$8\alpha_0\alpha_7+8\alpha_1\alpha_7+6\alpha_2\alpha_7+4\alpha_3\alpha_7+2\alpha_4\alpha_7+6\alpha_6\alpha_7+\alpha_7^2+11\alpha_0\alpha_8+12\alpha_1\alpha_8+9\alpha_2\alpha_8+6\alpha_3\alpha_8+3\alpha_4\alpha_8+8\alpha_6\alpha_8+$
$4\alpha_7\alpha_8+3\alpha_8^2))+p^3(-1+q)q^2(-36\alpha_0\alpha_1^2-20\alpha_1^3-54\alpha_0\alpha_1\alpha_2-45\alpha_1^2\alpha_2-18\alpha_0\alpha_2^2-33\alpha_1\alpha_2^2-8\alpha_2^3-36\alpha_0\alpha_1\alpha_3-$
$30\alpha_1^2\alpha_3-24\alpha_0\alpha_2\alpha_3-44\alpha_1\alpha_2\alpha_3-16\alpha_2^2\alpha_3-6\alpha_0\alpha_3^2-14\alpha_1\alpha_3^2-10\alpha_2\alpha_3^2-2\alpha_3^3-18\alpha_0\alpha_1\alpha_4-15\alpha_1^2\alpha_4-$
$12\alpha_0\alpha_2\alpha_4-22\alpha_1\alpha_2\alpha_4-8\alpha_2^2\alpha_4-6\alpha_0\alpha_3\alpha_4-14\alpha_1\alpha_3\alpha_4-10\alpha_2\alpha_3\alpha_4-3\alpha_3^2\alpha_4-3\alpha_1\alpha_4^2-2\alpha_2\alpha_4^2-\alpha_3\alpha_4^2-$
$24\alpha_1^2\alpha_6-36\alpha_1\alpha_2\alpha_6-12\alpha_2^2\alpha_6-24\alpha_1\alpha_3\alpha_6-16\alpha_2\alpha_3\alpha_6-4\alpha_3^2\alpha_6-12\alpha_1\alpha_4\alpha_6-8\alpha_2\alpha_4\alpha_6-4\alpha_3\alpha_4\alpha_6-12\alpha_1^2\alpha_7-$
$18\alpha_1\alpha_2\alpha_7-6\alpha_2^2\alpha_7-12\alpha_1\alpha_3\alpha_7-8\alpha_2\alpha_3\alpha_7-2\alpha_3^2\alpha_7-6\alpha_1\alpha_4\alpha_7-4\alpha_2\alpha_4\alpha_7-2\alpha_3\alpha_4\alpha_7-18\alpha_1^2\alpha_8-27\alpha_1\alpha_2\alpha_8-$
$9\alpha_2^2\alpha_8-18\alpha_1\alpha_3\alpha_8-12\alpha_2\alpha_3\alpha_8-3\alpha_3^2\alpha_8-9\alpha_1\alpha_4\alpha_8-6\alpha_2\alpha_4\alpha_8-3\alpha_3\alpha_4\alpha_8+q^3(2\alpha_0+\alpha_8)(10\alpha_0^2+10\alpha_0\alpha_8+$
$\alpha_8^2)+q(-20\alpha_0^3-96\alpha_0^2\alpha_1-4\alpha_0\alpha_1^2+20\alpha_1^3-72\alpha_0^2\alpha_2-6\alpha_0\alpha_1\alpha_2+45\alpha_1^2\alpha_2-6\alpha_0\alpha_2^2+33\alpha_1\alpha_2^2+8\alpha_2^3-48\alpha_0^2\alpha_3-$
$4\alpha_0\alpha_1\alpha_3+30\alpha_1^2\alpha_3-8\alpha_0\alpha_2\alpha_3+44\alpha_1\alpha_2\alpha_3+16\alpha_2^2\alpha_3-6\alpha_0\alpha_3^2+14\alpha_1\alpha_3^2+10\alpha_2\alpha_3^2+2\alpha_3^3-24\alpha_0^2\alpha_4-2\alpha_0\alpha_1\alpha_4+$
$15\alpha_1^2\alpha_4-4\alpha_0\alpha_2\alpha_4+22\alpha_1\alpha_2\alpha_4+8\alpha_2^2\alpha_4-6\alpha_0\alpha_3\alpha_4+14\alpha_1\alpha_3\alpha_4+10\alpha_2\alpha_3\alpha_4+3\alpha_3^2\alpha_4-4\alpha_0\alpha_4^2+3\alpha_1\alpha_4^2+2\alpha_2\alpha_4^2+$
$\alpha_3\alpha_4^2-40\alpha_0^2\alpha_6-64\alpha_0\alpha_1\alpha_6+24\alpha_1^2\alpha_6-48\alpha_0\alpha_2\alpha_6+36\alpha_1\alpha_2\alpha_6+12\alpha_2^2\alpha_6-32\alpha_0\alpha_3\alpha_6+24\alpha_1\alpha_3\alpha_6+16\alpha_2\alpha_3\alpha_6+$
$4\alpha_3^2\alpha_6-16\alpha_0\alpha_4\alpha_6+12\alpha_1\alpha_4\alpha_6+8\alpha_2\alpha_4\alpha_6+4\alpha_3\alpha_4\alpha_6-24\alpha_0\alpha_6^2-4\alpha_6^3-20\alpha_0^2\alpha_7-32\alpha_0\alpha_1\alpha_7+12\alpha_1^2\alpha_7-$
$24\alpha_0\alpha_2\alpha_7+18\alpha_1\alpha_2\alpha_7+6\alpha_2^2\alpha_7-16\alpha_0\alpha_3\alpha_7+12\alpha_1\alpha_3\alpha_7+8\alpha_2\alpha_3\alpha_7+2\alpha_3^2\alpha_7-8\alpha_0\alpha_4\alpha_7+6\alpha_1\alpha_4\alpha_7+4\alpha_2\alpha_4\alpha_7+$
$2\alpha_3\alpha_4\alpha_7-24\alpha_0\alpha_6\alpha_7-6\alpha_6^2\alpha_7-4\alpha_0\alpha_7^2-2\alpha_6\alpha_7^2-30\alpha_0^2\alpha_8-96\alpha_0\alpha_1\alpha_8-2\alpha_1^2\alpha_8-72\alpha_0\alpha_2\alpha_8-3\alpha_1\alpha_2\alpha_8-3\alpha_2^2\alpha_8-$
$48\alpha_0\alpha_3\alpha_8-2\alpha_1\alpha_3\alpha_8-4\alpha_2\alpha_3\alpha_8-3\alpha_3^2\alpha_8-24\alpha_0\alpha_4\alpha_8-\alpha_1\alpha_4\alpha_8-2\alpha_2\alpha_4\alpha_8-3\alpha_3\alpha_4\alpha_8-2\alpha_4^2\alpha_8-40\alpha_0\alpha_6\alpha_8-$
$32\alpha_1\alpha_6\alpha_8-24\alpha_2\alpha_6\alpha_8-16\alpha_3\alpha_6\alpha_8-8\alpha_4\alpha_6\alpha_8-12\alpha_6^2\alpha_8-20\alpha_0\alpha_7\alpha_8-16\alpha_1\alpha_7\alpha_8-12\alpha_2\alpha_7\alpha_8-8\alpha_3\alpha_7\alpha_8-$
$4\alpha_4\alpha_7\alpha_8-12\alpha_6\alpha_7\alpha_8-2\alpha_7^2\alpha_8-18\alpha_0\alpha_8^2-24\alpha_1\alpha_8^2-18\alpha_2\alpha_8^2-12\alpha_3\alpha_8^2-6\alpha_4\alpha_8^2-12\alpha_6\alpha_8^2-6\alpha_7\alpha_8^2-4\alpha_8^3)+$
$q^2(96\alpha_0^2\alpha_1+40\alpha_0\alpha_1^2+72\alpha_0^2\alpha_2+60\alpha_0\alpha_1\alpha_2+24\alpha_0\alpha_2^2+48\alpha_0^2\alpha_3+40\alpha_0\alpha_1\alpha_3+32\alpha_0\alpha_2\alpha_3+12\alpha_0\alpha_3^2+24\alpha_0^2\alpha_4+$
$20\alpha_0\alpha_1\alpha_4+16\alpha_0\alpha_2\alpha_4+12\alpha_0\alpha_3\alpha_4+4\alpha_0\alpha_4^2+40\alpha_0^2\alpha_6+64\alpha_0\alpha_1\alpha_6+48\alpha_0\alpha_2\alpha_6+32\alpha_0\alpha_3\alpha_6+16\alpha_0\alpha_4\alpha_6+$
$24\alpha_0\alpha_6^2+20\alpha_0^2\alpha_7+32\alpha_0\alpha_1\alpha_7+24\alpha_0\alpha_2\alpha_7+16\alpha_0\alpha_3\alpha_7+8\alpha_0\alpha_4\alpha_7+24\alpha_0\alpha_6\alpha_7+4\alpha_0\alpha_7^2+96\alpha_0\alpha_1\alpha_8+20\alpha_1^2\alpha_8+$
$72\alpha_0\alpha_2\alpha_8+30\alpha_1\alpha_2\alpha_8+12\alpha_2^2\alpha_8+48\alpha_0\alpha_3\alpha_8+20\alpha_1\alpha_3\alpha_8+16\alpha_2\alpha_3\alpha_8+6\alpha_3^2\alpha_8+24\alpha_0\alpha_4\alpha_8+10\alpha_1\alpha_4\alpha_8+$
$8\alpha_2\alpha_4\alpha_8+6\alpha_3\alpha_4\alpha_8+2\alpha_4^2\alpha_8+40\alpha_0\alpha_6\alpha_8+32\alpha_1\alpha_6\alpha_8+24\alpha_2\alpha_6\alpha_8+16\alpha_3\alpha_6\alpha_8+8\alpha_4\alpha_6\alpha_8+12\alpha_6^2\alpha_8+$
$20\alpha_0\alpha_7\alpha_8+16\alpha_1\alpha_7\alpha_8+12\alpha_2\alpha_7\alpha_8+8\alpha_3\alpha_7\alpha_8+4\alpha_4\alpha_7\alpha_8+12\alpha_6\alpha_7\alpha_8+2\alpha_7^2\alpha_8+6\alpha_0\alpha_8^2+24\alpha_1\alpha_8^2+18\alpha_2\alpha_8^2+$
$12\alpha_3\alpha_8^2+6\alpha_4\alpha_8^2+12\alpha_6\alpha_8^2+6\alpha_7\alpha_8^2+3\alpha_8^3))+p^2q(-24\alpha_0\alpha_1^3-15\alpha_1^4-54\alpha_0\alpha_1^2\alpha_2-45\alpha_1^3\alpha_2-36\alpha_0\alpha_1\alpha_2^2-48\alpha_1^2\alpha_2^2-$
$6\alpha_0\alpha_2^3-21\alpha_1\alpha_2^3-3\alpha_2^4-36\alpha_0\alpha_1^2\alpha_3-30\alpha_1^3\alpha_3-48\alpha_0\alpha_1\alpha_2\alpha_3-64\alpha_1^2\alpha_2\alpha_3-12\alpha_0\alpha_2^2\alpha_3-42\alpha_1\alpha_2^2\alpha_3-8\alpha_2^3\alpha_3-$
$12\alpha_0\alpha_1\alpha_3^2-19\alpha_1^2\alpha_3^2-6\alpha_0\alpha_2\alpha_3^2-25\alpha_1\alpha_2\alpha_3^2-7\alpha_2^2\alpha_3^2-4\alpha_1\alpha_3^3-2\alpha_2\alpha_3^3-18\alpha_0\alpha_1^2\alpha_4-15\alpha_1^3\alpha_4-24\alpha_0\alpha_1\alpha_2\alpha_4-$
$32\alpha_1^2\alpha_2\alpha_4-6\alpha_0\alpha_2^2\alpha_4-21\alpha_1\alpha_2^2\alpha_4-4\alpha_2^3\alpha_4-12\alpha_0\alpha_1\alpha_3\alpha_4-19\alpha_1^2\alpha_3\alpha_4-6\alpha_0\alpha_2\alpha_3\alpha_4-25\alpha_1\alpha_2\alpha_3\alpha_4-$
$7\alpha_2^2\alpha_3\alpha_4-6\alpha_1\alpha_3^2\alpha_4-3\alpha_2\alpha_3^2\alpha_4-3\alpha_1^2\alpha_4^2-4\alpha_1\alpha_2\alpha_4^2-\alpha_2^2\alpha_4^2-2\alpha_1\alpha_3\alpha_4^2-\alpha_2\alpha_3\alpha_4^2-16\alpha_1^3\alpha_6-36\alpha_1^2\alpha_2\alpha_6-$
$24\alpha_1\alpha_2^2\alpha_6-4\alpha_2^3\alpha_6-24\alpha_1^2\alpha_3\alpha_6-32\alpha_1\alpha_2\alpha_3\alpha_6-8\alpha_2^2\alpha_3\alpha_6-8\alpha_1\alpha_3^2\alpha_6-4\alpha_2\alpha_3^2\alpha_6-12\alpha_1^2\alpha_4\alpha_6-16\alpha_1\alpha_2\alpha_4\alpha_6-$
$4\alpha_2^2\alpha_4\alpha_6-8\alpha_1\alpha_3\alpha_4\alpha_6-4\alpha_2\alpha_3\alpha_4\alpha_6-8\alpha_1^3\alpha_7-18\alpha_1^2\alpha_2\alpha_7-12\alpha_1\alpha_2^2\alpha_7-2\alpha_2^3\alpha_7-12\alpha_1^2\alpha_3\alpha_7-16\alpha_1\alpha_2\alpha_3\alpha_7-$
$4\alpha_2^2\alpha_3\alpha_7-4\alpha_1\alpha_3^2\alpha_7-2\alpha_2\alpha_3^2\alpha_7-6\alpha_1^2\alpha_4\alpha_7-8\alpha_1\alpha_2\alpha_4\alpha_7-2\alpha_2^2\alpha_4\alpha_7-4\alpha_1\alpha_3\alpha_4\alpha_7-2\alpha_2\alpha_3\alpha_4\alpha_7-12\alpha_1^3\alpha_8-$
$27\alpha_1^2\alpha_2\alpha_8-18\alpha_1\alpha_2^2\alpha_8-3\alpha_2^3\alpha_8-18\alpha_1^2\alpha_3\alpha_8-24\alpha_1\alpha_2\alpha_3\alpha_8-6\alpha_2^2\alpha_3\alpha_8-6\alpha_1\alpha_3^2\alpha_8-3\alpha_2\alpha_3^2\alpha_8-9\alpha_1^2\alpha_4\alpha_8-$
$12\alpha_1\alpha_2\alpha_4\alpha_8-3\alpha_2^2\alpha_4\alpha_8-6\alpha_1\alpha_3\alpha_4\alpha_8-3\alpha_2\alpha_3\alpha_4\alpha_8+3q^4\alpha_0(\alpha_0+\alpha_8)(5\alpha_0^2+5\alpha_0\alpha_8+\alpha_8^2)+q^2(-105\alpha_0^4-$
$288\alpha_0^3\alpha_1-120\alpha_0^2\alpha_1^2-24\alpha_0\alpha_1^3-15\alpha_1^4-216\alpha_0^3\alpha_2-180\alpha_0^2\alpha_1\alpha_2-54\alpha_0\alpha_1^2\alpha_2-45\alpha_1^3\alpha_2-72\alpha_0^2\alpha_2^2-36\alpha_0\alpha_1\alpha_2^2-$
$48\alpha_1^2\alpha_2^2-6\alpha_0\alpha_2^3-21\alpha_1\alpha_2^3-3\alpha_2^4-144\alpha_0^3\alpha_3-120\alpha_0^2\alpha_1\alpha_3-36\alpha_0\alpha_1^2\alpha_3-30\alpha_1^3\alpha_3-96\alpha_0^2\alpha_2\alpha_3-48\alpha_0\alpha_1\alpha_2\alpha_3-$
$64\alpha_1^2\alpha_2\alpha_3-12\alpha_0\alpha_2^2\alpha_3-42\alpha_1\alpha_2^2\alpha_3-8\alpha_2^3\alpha_3-36\alpha_0^2\alpha_3^2-12\alpha_0\alpha_1\alpha_3^2-19\alpha_1^2\alpha_3^2-6\alpha_0\alpha_2\alpha_3^2-25\alpha_1\alpha_2\alpha_3^2-7\alpha_2^2\alpha_3^2-$
$4\alpha_1\alpha_3^3-2\alpha_2\alpha_3^3-72\alpha_0^3\alpha_4-60\alpha_0^2\alpha_1\alpha_4-18\alpha_0\alpha_1^2\alpha_4-15\alpha_1^3\alpha_4-48\alpha_0^2\alpha_2\alpha_4-24\alpha_0\alpha_1\alpha_2\alpha_4-32\alpha_1^2\alpha_2\alpha_4-$

$$6\alpha_0\alpha_2^2\alpha_4 - 21\alpha_1\alpha_2^2\alpha_4 - 4\alpha_2^3\alpha_4 - 36\alpha_0^2\alpha_3\alpha_4 - 12\alpha_0\alpha_1\alpha_3\alpha_4 - 19\alpha_1^2\alpha_3\alpha_4 - 6\alpha_0\alpha_2\alpha_3\alpha_4 - 25\alpha_1\alpha_2\alpha_3\alpha_4 -$$
$$7\alpha_2^2\alpha_3\alpha_4 - 6\alpha_1\alpha_3^2\alpha_4 - 3\alpha_2\alpha_3^2\alpha_4 - 12\alpha_0^2\alpha_4^2 - 3\alpha_1^2\alpha_4^2 - 4\alpha_1\alpha_2\alpha_4^2 - \alpha_2^2\alpha_4^2 - 2\alpha_1\alpha_3\alpha_4^2 - \alpha_2\alpha_3\ \alpha_4^2 - 240\alpha_0^3\alpha_6 -$$
$$384\alpha_0^2\alpha_1\alpha_6 - 152\alpha_0\alpha_1^2\alpha_6 - 56\alpha_1^3\alpha_6 - 288\alpha_0^2\alpha_2\alpha_6 - 228\alpha_0\alpha_1\alpha_2\alpha_6 - 126\alpha_1^2\alpha_2\alpha_6 - 84\alpha_0\alpha_2^2\alpha_6 - 90\alpha_1\alpha_2^2\alpha_6 -$$
$$20\alpha_2^3\alpha_6 - 192\alpha_0^2\alpha_3\alpha_6 - 152\alpha_0\alpha_1\alpha_3\alpha_6 - 84\alpha_1^2\alpha_3\alpha_6 - 112\alpha_0\alpha_2\alpha_3\alpha_6 - 120\alpha_1\alpha_2\alpha_3\alpha_6 - 40\alpha_2^2\alpha_3\alpha_6 - 36\alpha_0\alpha_3^2\alpha_6 -$$
$$36\alpha_1\alpha_3^2\alpha_6 - 24\alpha_2\alpha_3^2\alpha_6 - 4\alpha_3^3\alpha_6 - 96\alpha_0^2\alpha_4\alpha_6 - 76\alpha_0\alpha_1\alpha_4\alpha_6 - 42\alpha_1^2\alpha_4\alpha_6 - 56\alpha_0\alpha_2\alpha_4\alpha_6 - 60\alpha_1\alpha_2\alpha_4\alpha_6 -$$
$$20\alpha_2^2\alpha_4\alpha_6 - 36\alpha_0\alpha_3\alpha_4\alpha_6 - 36\alpha_1\alpha_3\alpha_4\alpha_6 - 24\alpha_2\alpha_3\alpha_4\alpha_6 - 6\alpha_3^2\alpha_4\alpha_6 - 8\alpha_0\alpha_4^2\alpha_6 - 6\alpha_1\alpha_4^2\alpha_6 - 4\alpha_2\alpha_4^2\alpha_6 -$$
$$2\alpha_3\alpha_4^2\alpha_6 - 197\alpha_0^2\alpha_6^2 - 200\alpha_0\alpha_1\alpha_6^2 - 78\alpha_1^2\alpha_6^2 - 150\alpha_0\alpha_2\alpha_6^2 - 117\alpha_1\alpha_2\alpha_6^2 - 42\alpha_2^2\alpha_6^2 - 100\alpha_0\alpha_3\alpha_6^2 - 78\alpha_1\alpha_3\alpha_6^2 -$$
$$56\alpha_2\alpha_3\alpha_6^2 - 17\alpha_3^2\alpha_6^2 - 50\alpha_0\alpha_4\alpha_6^2 - 39\alpha_1\alpha_4\alpha_6^2 - 28\alpha_2\alpha_4\alpha_6^2 - 17\alpha_3\alpha_4\alpha_6^2 - 3\alpha_4^2\alpha_6^2 - 72\alpha_0\alpha_6^3 - 48\alpha_1\alpha_6^3 - 36\alpha_2\alpha_6^3 -$$
$$24\alpha_3\alpha_6^3 - 12\alpha_4\alpha_6^3 - 11\alpha_6^4 - 120\alpha_0^3\alpha_7 - 192\alpha_0^2\alpha_1\alpha_7 - 76\alpha_0\alpha_1^2\alpha_7 - 28\alpha_1^3\alpha_7 - 144\alpha_0^2\alpha_2\alpha_7 - 114\alpha_0\alpha_1\alpha_2\alpha_7 -$$
$$63\alpha_1^2\alpha_2\alpha_7 - 42\alpha_0\alpha_2^2\alpha_7 - 45\alpha_1\alpha_2^2\alpha_7 - 10\alpha_2^3\alpha_7 - 96\alpha_0^2\alpha_3\alpha_7 - 76\alpha_0\alpha_1\alpha_3\alpha_7 - 42\alpha_1^2\alpha_3\alpha_7 - 56\alpha_0\alpha_2\alpha_3\alpha_7 -$$
$$60\alpha_1\alpha_2\alpha_3\alpha_7 - 20\alpha_2^2\alpha_3\alpha_7 - 18\alpha_0\alpha_3^2\alpha_7 - 18\alpha_1\alpha_3^2\alpha_7 - 12\alpha_2\alpha_3^2\alpha_7 - 2\alpha_3^3\alpha_7 - 48\alpha_0^2\alpha_4\alpha_7 - 38\alpha_0\alpha_1\alpha_4\alpha_7 -$$
$$21\alpha_1^2\alpha_4\alpha_7 - 28\alpha_0\alpha_2\alpha_4\alpha_7 - 30\alpha_1\alpha_2\alpha_4\alpha_7 - 10\alpha_2^2\alpha_4\alpha_7 - 18\alpha_0\alpha_3\alpha_4\alpha_7 - 18\alpha_1\alpha_3\alpha_4\alpha_7 - 12\alpha_2\alpha_3\alpha_4\alpha_7 - 3\alpha_3^2\alpha_4\alpha_7 -$$
$$4\alpha_0\alpha_4^2\alpha_7 - 3\alpha_1\alpha_4^2\alpha_7 - 2\alpha_2\alpha_4^2\alpha_7 - \alpha_3\alpha_4^2\ \alpha_7 - 197\alpha_0^2\alpha_6\alpha_7 - 200\alpha_0\alpha_1\alpha_6\alpha_7 - 78\alpha_1^2\alpha_6\alpha_7 - 150\alpha_0\alpha_2\alpha_6\alpha_7 -$$
$$117\alpha_1\alpha_2\alpha_6\alpha_7 - 42\alpha_2^2\alpha_6\alpha_7 - 100\alpha_0\alpha_3\alpha_6\alpha_7 - 78\alpha_1\alpha_3\alpha_6\alpha_7 - 56\alpha_2\alpha_3\alpha_6\alpha_7 - 17\alpha_3^2\alpha_6\alpha_7 - 50\alpha_0\alpha_4\alpha_6\alpha_7 -$$
$$39\alpha_1\alpha_4\alpha_6\alpha_7 - 28\alpha_2\alpha_4\alpha_6\alpha_7 - 17\alpha_3\alpha_4\alpha_6\alpha_7 - 3\alpha_4^2\alpha_6\alpha_7 - 108\alpha_0\alpha_6^2\alpha_7 - 72\alpha_1\alpha_6^2\alpha_7 - 54\alpha_2\alpha_6^2\alpha_7 - 36\alpha_3\alpha_6^2\alpha_7 -$$
$$18\alpha_4\alpha_6^2\alpha_7 - 22\alpha_6^3\alpha_7 - 47\alpha_0^2\alpha_7^2 - 56\alpha_0\alpha_1\alpha_7^2 - 22\alpha_1^2\alpha_7^2 - 42\alpha_0\alpha_2\alpha_7^2 - 33\alpha_1\alpha_2\alpha_7^2 - 12\alpha_2^2\alpha_7^2 - 28\alpha_0\alpha_3\alpha_7^2 -$$
$$22\alpha_1\alpha_3\alpha_7^2 - 16\alpha_2\alpha_3\alpha_7^2 - 5\alpha_3^2\alpha_7^2 - 14\alpha_0\alpha_4\alpha_7^2 - 11\alpha_1\alpha_4\alpha_7^2 - 8\alpha_2\alpha_4\alpha_7^2 - 5\alpha_3\alpha_4\alpha_7^2 - \alpha_4^2\alpha_7^2 - 56\alpha_0\alpha_6\alpha_7^2 -$$
$$40\alpha_1\alpha_6\alpha_7^2 - 30\alpha_2\alpha_6\alpha_7^2 - 20\alpha_3\alpha_6\alpha_7^2 - 10\alpha_4\alpha_6\alpha_7^2 - 18\alpha_6^2\alpha_7^2 - 10\alpha_0\alpha_7^3 - 8\alpha_1\alpha_7^3 - 6\alpha_2\alpha_7^3 - 4\alpha_3\alpha_7^3 - 2\alpha_4\alpha_7^3 - 7\alpha_6\alpha_7^3 -$$
$$\alpha_7^4 - 210\alpha_0^3\alpha_8 - 432\alpha_0^2\alpha_1\alpha_8 - 120\alpha_0\alpha_1^2\alpha_8 - 12\alpha_1^3\alpha_8 - 324\alpha_0^2\alpha_2\alpha_8 - 180\alpha_0\alpha_1\alpha_2\alpha_8 - 27\alpha_1^2\alpha_2\alpha_8 - 72\alpha_0\alpha_2^2\alpha_8 -$$
$$18\alpha_1\alpha_2^2\alpha_8 - 3\alpha_2^3\alpha_8 - 216\alpha_0^2\alpha_3\alpha_8 - 120\alpha_0\alpha_1\alpha_3\alpha_8 - 18\alpha_1^2\alpha_3\alpha_8 - 96\alpha_0\alpha_2\alpha_3\alpha_8 - 24\alpha_1\alpha_2\alpha_3\alpha_8 - 6\alpha_2^2\alpha_3\alpha_8 -$$
$$36\alpha_0\alpha_3^2\alpha_8 - 6\alpha_1\alpha_3^2\alpha_8 - 3\alpha_2\alpha_3^2\alpha_8 - 108\alpha_0^2\alpha_4\alpha_8 - 60\alpha_0\alpha_1\alpha_4\alpha_8 - 9\alpha_1^2\alpha_4\alpha_8 - 48\alpha_0\alpha_2\alpha_4\alpha_8 - 12\alpha_1\alpha_2\alpha_4\alpha_8 -$$
$$3\alpha_2^2\alpha_4\alpha_8 - 36\alpha_0\alpha_3\alpha_4\alpha_8 - 6\alpha_1\alpha_3\alpha_4\alpha_8 - 3\alpha_2\alpha_3\alpha_4\alpha_8 - 12\alpha_0\alpha_4^2\alpha_8 - 360\alpha_0^2\alpha_6\alpha_8 - 384\alpha_0\alpha_1\alpha_6\alpha_8 - 76\alpha_1^2\alpha_6\alpha_8 -$$
$$288\alpha_0\alpha_2\alpha_6\alpha_8 - 114\alpha_1\alpha_2\alpha_6\alpha_8 - 42\alpha_2^2\alpha_6\alpha_8 - 192\alpha_0\alpha_3\alpha_6\alpha_8 - 76\alpha_1\alpha_3\alpha_6\alpha_8 - 56\alpha_2\alpha_3\alpha_6\alpha_8 - 18\alpha_3^2\alpha_6\alpha_8 -$$
$$96\alpha_0\alpha_4\alpha_6\alpha_8 - 38\alpha_1\alpha_4\alpha_6\alpha_8 - 28\alpha_2\alpha_4\alpha_6\alpha_8 - 18\alpha_3\alpha_4\alpha_6\alpha_8 - 4\alpha_4^2\alpha_6\alpha_8 - 197\alpha_0\alpha_6^2\alpha_8 - 100\alpha_1\alpha_6^2\alpha_8 - 75\alpha_2\alpha_6^2\alpha_8 -$$
$$50\alpha_3\alpha_6^2\alpha_8 - 25\alpha_4\alpha_6^2\alpha_8 - 36\alpha_6^3\alpha_8 - 180\alpha_0^2\alpha_7\alpha_8 - 192\alpha_0\alpha_1\alpha_7\alpha_8 - 38\alpha_1^2\alpha_7\alpha_8 - 144\alpha_0\alpha_2\alpha_7\alpha_8 - 57\alpha_1\alpha_2\alpha_7\alpha_8 -$$
$$21\alpha_2^2\alpha_7\alpha_8 - 96\alpha_0\alpha_3\alpha_7\alpha_8 - 38\alpha_1\alpha_3\alpha_7\alpha_8 - 28\alpha_2\alpha_3\alpha_7\alpha_8 - 9\alpha_3^2\alpha_7\alpha_8 - 48\alpha_0\alpha_4\alpha_7\alpha_8 - 19\alpha_1\alpha_4\alpha_7\alpha_8 - 14\alpha_2\alpha_4\alpha_7\alpha_8 -$$
$$9\alpha_3\alpha_4\alpha_7\alpha_8 - 2\alpha_4^2\alpha_7\alpha_8 - 197\alpha_0\alpha_6\alpha_7\alpha_8 - 100\alpha_1\alpha_6\alpha_7\alpha_8 - 75\alpha_2\alpha_6\alpha_7\alpha_8 - 50\alpha_3\alpha_6\alpha_7\alpha_8 - 25\alpha_4\alpha_6\alpha_7\alpha_8 -$$
$$54\alpha_6^2\alpha_7\alpha_8 - 47\alpha_0\alpha_7^2\alpha_8 - 28\alpha_1\alpha_7^2\alpha_8 - 21\alpha_2\alpha_7^2\alpha_8 - 14\alpha_3\alpha_7^2\alpha_8 - 7\alpha_4\alpha_7^2\alpha_8 - 28\alpha_6\alpha_7^2\alpha_8 - 5\alpha_7^3\alpha_8 - 156\alpha_0^2\alpha_8^2 -$$
$$192\alpha_0\alpha_1\alpha_8^2 - 20\alpha_1^2\alpha_8^2 - 144\alpha_0\alpha_2\alpha_8^2 - 30\alpha_1\alpha_2\alpha_8^2 - 12\alpha_2^2\alpha_8^2 - 96\alpha_0\alpha_3\alpha_8^2 - 20\alpha_1\alpha_3\alpha_8^2 - 16\alpha_2\alpha_3\alpha_8^2 - 6\alpha_3^2\alpha_8^2 -$$
$$48\alpha_0\alpha_4\alpha_8^2 - 10\alpha_1\alpha_4\alpha_8^2 - 8\alpha_2\alpha_4\alpha_8^2 - 6\alpha_3\alpha_4\alpha_8^2 - 2\alpha_4^2\alpha_8^2 - 172\alpha_0\alpha_6\alpha_8^2 - 80\alpha_1\alpha_6\alpha_8^2 - 60\alpha_2\alpha_6\alpha_8^2 - 40\alpha_3\alpha_6\alpha_8^2 -$$
$$20\alpha_4\alpha_6\alpha_8^2 - 45\alpha_6^2\alpha_8^2 - 86\alpha_0\alpha_7\alpha_8^2 - 40\alpha_1\alpha_7\alpha_8^2 - 30\alpha_2\alpha_7\alpha_8^2 - 20\alpha_3\alpha_7\alpha_8^2 - 10\alpha_4\alpha_7\alpha_8^2 - 45\alpha_6\alpha_7\alpha_8^2 - 11\alpha_7^2\alpha_8^2 -$$
$$51\alpha_0\alpha_8^3 - 24\alpha_1\alpha_8^3 - 18\alpha_2\alpha_8^3 - 12\alpha_3\alpha_8^3 - 6\alpha_4\alpha_8^3 - 26\alpha_6\alpha_8^3 - 13\alpha_7\alpha_8^3 - 6\alpha_8^4) + q^3(30\alpha_0^4 + 144\alpha_0^3\alpha_1 + 60\alpha_0^2\alpha_1^2 +$$
$$108\alpha_0^3\alpha_2 + 90\alpha_0^2\alpha_1\alpha_2 + 36\alpha_0^2\alpha_2^2 + 72\alpha_0^3\alpha_3 + 60\alpha_0^2\alpha_1\alpha_3 + 48\alpha_0^2\alpha_2\alpha_3 + 18\alpha_0^2\alpha_3^2 + 36\alpha_0^3\alpha_4 + 30\alpha_0^2\alpha_1\alpha_4 +$$
$$24\alpha_0^2\alpha_2\alpha_4 + 18\alpha_0^2\alpha_3\alpha_4 + 6\alpha_0^2\alpha_4^2 + 80\alpha_0^3\alpha_6 + 96\alpha_0^2\alpha_1\alpha_6 + 72\alpha_0^2\alpha_2\alpha_6 + 48\alpha_0^2\alpha_3\alpha_6 + 24\alpha_0^2\alpha_4\alpha_6 + 36\alpha_0^2\alpha_6^2 +$$
$$40\alpha_0^3\alpha_7 + 48\alpha_0^2\alpha_1\alpha_7 + 36\alpha_0^2\alpha_2\alpha_7 + 24\alpha_0^2\alpha_3\alpha_7 + 12\alpha_0^2\alpha_4\alpha_7 + 36\alpha_0^2\alpha_6\alpha_7 + 6\alpha_0^2\alpha_7^2 + 60\alpha_0^3\alpha_8 + 216\alpha_0^2\alpha_1\alpha_8 +$$
$$60\alpha_0\alpha_1^2\alpha_8 + 162\alpha_0^2\alpha_2\alpha_8 + 90\alpha_0\alpha_1\alpha_2\alpha_8 + 36\alpha_0\alpha_2^2\alpha_8 + 108\alpha_0^2\alpha_3\alpha_8 + 60\alpha_0\alpha_1\alpha_3\alpha_8 + 48\alpha_0\alpha_2\alpha_3\alpha_8 + 18\alpha_0\alpha_3^2\alpha_8 +$$
$$54\alpha_0^2\alpha_4\alpha_8 + 30\alpha_0\alpha_1\alpha_4\alpha_8 + 24\alpha_0\alpha_2\alpha_4\alpha_8 + 18\alpha_0\alpha_3\alpha_4\alpha_8 + 6\alpha_0\alpha_4^2\alpha_8 + 120\alpha_0^2\alpha_6\alpha_8 + 96\alpha_0\alpha_1\alpha_6\alpha_8 + 72\alpha_0\alpha_2\alpha_6\alpha_8 +$$
$$48\alpha_0\alpha_3\alpha_6\alpha_8 + 24\alpha_0\alpha_4\alpha_6\alpha_8 + 36\alpha_0\alpha_6^2\alpha_8 + 60\alpha_0^2\alpha_7\alpha_8 + 48\alpha_0\alpha_1\alpha_7\alpha_8 + 36\alpha_0\alpha_2\alpha_7\alpha_8 + 24\alpha_0\alpha_3\alpha_7\alpha_8 + 12\alpha_0\alpha_4\alpha_7\alpha_8 +$$
$$36\alpha_0\alpha_6\alpha_7\alpha_8 + 6\alpha_0\alpha_7^2\alpha_8 + 51\alpha_0^2\alpha_8^2 + 96\alpha_0\alpha_1\alpha_8^2 + 10\alpha_1^2\alpha_8^2 + 72\alpha_0\alpha_2\alpha_8^2 + 15\alpha_1\alpha_2\alpha_8^2 + 6\alpha_2^2\alpha_8^2 + 48\alpha_0\alpha_3\alpha_8^2 +$$
$$10\alpha_1\alpha_3\alpha_8^2 + 8\alpha_2\alpha_3\alpha_8^2 + 3\alpha_3^2\alpha_8^2 + 24\alpha_0\alpha_4\alpha_8^2 + 5\alpha_1\alpha_4\alpha_8^2 + 4\alpha_2\alpha_4\alpha_8^2 + 3\alpha_3\alpha_4\alpha_8^2 + \alpha_4^2\alpha_8^2 + 56\alpha_0\alpha_6\alpha_8^2 + 16\alpha_1\alpha_6\alpha_8^2 +$$
$$12\alpha_2\alpha_6\alpha_8^2 + 8\alpha_3\alpha_6\alpha_8^2 + 4\alpha_4\alpha_6\alpha_8^2 + 6\alpha_6^2\alpha_8^2 + 28\alpha_0\alpha_7\alpha_8^2 + 8\alpha_1\alpha_7\alpha_8^2 + 6\alpha_2\alpha_7\alpha_8^2 + 4\alpha_3\alpha_7\alpha_8^2 + 2\alpha_4\alpha_7\alpha_8^2 +$$
$$6\alpha_6\alpha_7\alpha_8^2 + \alpha_7^2\alpha_8^2 + 21\alpha_0\alpha_8^3 + 12\alpha_1\alpha_8^3 + 9\alpha_2\alpha_8^3 + 6\alpha_3\alpha_8^3 + 3\alpha_4\alpha_8^3 + 8\alpha_6\alpha_8^3 + 4\alpha_7\alpha_8^3 + 3\alpha_8^4) + q(60\alpha_0^4 + 144\alpha_0^3\alpha_1 +$$
$$60\alpha_0^2\alpha_1^2 + 48\alpha_0\alpha_1^3 + 30\alpha_1^4 + 108\alpha_0^3\alpha_2 + 90\alpha_0^2\alpha_1\alpha_2 + 108\alpha_0\alpha_1^2\alpha_2 + 90\alpha_1^3\alpha_2 + 36\alpha_0^2\alpha_2^2 + 72\alpha_0\alpha_1\alpha_2^2 + 96\alpha_1^2\alpha_2^2 +$$
$$12\alpha_0\alpha_2^3 + 42\alpha_1\alpha_2^3 + 6\alpha_2^4 + 72\alpha_0^3\alpha_3 + 60\alpha_0^2\alpha_1\alpha_3 + 72\alpha_0\alpha_1^2\alpha_3 + 60\alpha_1^3\alpha_3 + 48\alpha_0^2\alpha_2\alpha_3 + 96\alpha_0\alpha_1\alpha_2\alpha_3 + 128\alpha_1^2\alpha_2\alpha_3 +$$
$$24\alpha_0\alpha_2^2\alpha_3 + 84\alpha_1\alpha_2^2\alpha_3 + 16\alpha_2^3\alpha_3 + 18\alpha_0^2\alpha_3^2 + 24\alpha_0\alpha_1\alpha_3^2 + 38\alpha_1^2\alpha_3^2 + 12\alpha_0\alpha_2\alpha_3^2 + 50\alpha_1\alpha_2\alpha_3^2 + 14\alpha_2^2\alpha_3^2 +$$
$$8\alpha_1\alpha_3^3 + 4\alpha_2\alpha_3^3 + 36\alpha_0^3\alpha_4 + 30\alpha_0^2\alpha_1\alpha_4 + 36\alpha_0\alpha_1^2\alpha_4 + 30\alpha_1^3\alpha_4 + 24\alpha_0^2\alpha_2\alpha_4 + 48\alpha_0\alpha_1\alpha_2\alpha_4 + 64\alpha_1^2\alpha_2\alpha_4 +$$
$$12\alpha_0\alpha_2^2\alpha_4 + 42\alpha_1\alpha_2^2\alpha_4 + 8\alpha_2^3\alpha_4 + 18\alpha_0^2\alpha_3\alpha_4 + 24\alpha_0\alpha_1\alpha_3\alpha_4 + 38\alpha_1^2\alpha_3\alpha_4 + 12\alpha_0\alpha_2\alpha_3\alpha_4 + 50\alpha_1\alpha_2\alpha_3\alpha_4 +$$
$$14\alpha_2^2\alpha_3\alpha_4 + 12\alpha_1\alpha_3^2\alpha_4 + 6\alpha_2\alpha_3^2\alpha_4 + 6\alpha_0^2\alpha_4^2 + 6\alpha_1^2\alpha_4^2 + 8\alpha_1\alpha_2\alpha_4^2 + 2\alpha_2^2\alpha_4^2 + 4\alpha_1\alpha_3\alpha_4^2 + 2\alpha_2\alpha_3\alpha_4^2 + 160\alpha_0^3\alpha_6 +$$
$$288\alpha_0^2\alpha_1\alpha_6 + 152\alpha_0\alpha_1^2\alpha_6 + 72\alpha_1^3\alpha_6 + 216\alpha_0^2\alpha_2\alpha_6 + 228\alpha_0\alpha_1\alpha_2\alpha_6 + 162\alpha_1^2\alpha_2\alpha_6 + 84\alpha_0\alpha_2^2\alpha_6 + 114\alpha_1\alpha_2^2\alpha_6 +$$
$$24\alpha_2^3\alpha_6 + 144\alpha_0^2\alpha_3\alpha_6 + 152\alpha_0\alpha_1\alpha_3\alpha_6 + 108\alpha_1^2\alpha_3\alpha_6 + 112\alpha_0\alpha_2\alpha_3\alpha_6 + 152\alpha_1\alpha_2\alpha_3\alpha_6 + 48\alpha_2^2\alpha_3\alpha_6 + 36\alpha_0\alpha_3^2\alpha_6 +$$
$$44\alpha_1\alpha_3^2\alpha_6 + 28\alpha_2\alpha_3^2\alpha_6 + 4\alpha_3^3\alpha_6 + 72\alpha_0^2\alpha_4\alpha_6 + 76\alpha_0\alpha_1\alpha_4\alpha_6 + 54\alpha_1^2\alpha_4\alpha_6 + 56\alpha_0\alpha_2\alpha_4\alpha_6 + 76\alpha_1\alpha_2\alpha_4\alpha_6 +$$
$$24\alpha_2^2\alpha_4\alpha_6 + 36\alpha_0\alpha_3\alpha_4\alpha_6 + 44\alpha_1\alpha_3\alpha_4\alpha_6 + 28\alpha_2\alpha_3\alpha_4\alpha_6 + 6\alpha_3^2\alpha_4\alpha_6 + 8\alpha_0\alpha_4^2\alpha_6 + 6\alpha_1\alpha_4^2\alpha_6 + 4\alpha_2\alpha_4^2\alpha_6 +$$
$$2\alpha_3\alpha_4^2\alpha_6 + 161\alpha_0^2\alpha_6^2 + 200\alpha_0\alpha_1\alpha_6^2 + 78\alpha_1^2\alpha_6^2 + 150\alpha_0\alpha_2\alpha_6^2 + 117\alpha_1\alpha_2\alpha_6^2 + 42\alpha_2^2\alpha_6^2 + 100\alpha_0\alpha_3\alpha_6^2 + 78\alpha_1\alpha_3\alpha_6^2 +$$
$$56\alpha_2\alpha_3\alpha_6^2 + 17\alpha_3^2\alpha_6^2 + 50\alpha_0\alpha_4\alpha_6^2 + 39\alpha_1\alpha_4\alpha_6^2 + 28\alpha_2\alpha_4\alpha_6^2 + 17\alpha_3\alpha_4\alpha_6^2 + 3\alpha_4^2\alpha_6^2 + 72\alpha_0\alpha_6^3 + 48\alpha_1\alpha_6^3 + 36\alpha_2\alpha_6^3 +$$

$24\alpha_3\alpha_6^3 + 12\alpha_4\alpha_6^3 + 12\alpha_6^4 + 80\alpha_0^3\alpha_7 + 144\alpha_0^2\alpha_1\alpha_7 + 76\alpha_0\alpha_1^2\alpha_7 + 36\alpha_1^3\alpha_7 + 108\alpha_0^2\alpha_2\alpha_7 + 114\alpha_0\alpha_1\alpha_2\alpha_7 +$
$81\alpha_1^2\alpha_2\alpha_7 + 42\alpha_0\alpha_2^2\alpha_7 + 57\alpha_1\alpha_2^2\alpha_7 + 12\alpha_2^3\alpha_7 + 72\alpha_0^2\alpha_3\alpha_7 + 76\alpha_0\alpha_1\alpha_3\alpha_7 + 54\alpha_1^2\alpha_3\alpha_7 + 56\alpha_0\alpha_2\alpha_3\alpha_7 +$
$76\alpha_1\alpha_2\alpha_3\alpha_7 + 24\alpha_2^2\alpha_3\alpha_7 + 18\alpha_0\alpha_3^2\alpha_7 + 22\alpha_1\alpha_3^2\alpha_7 + 14\alpha_2\alpha_3^2\alpha_7 + 2\alpha_3^3\alpha_7 + 36\alpha_0^2\alpha_4\alpha_7 + 38\alpha_0\alpha_1\alpha_4\alpha_7 +$
$27\alpha_1^2\alpha_4\alpha_7+28\alpha_0\alpha_2\alpha_4\alpha_7+38\alpha_1\alpha_2\alpha_4\alpha_7+12\alpha_2^2\alpha_4\alpha_7+18\alpha_0\alpha_3\alpha_4\alpha_7+22\alpha_1\alpha_3\alpha_4\alpha_7+14\alpha_2\alpha_3\alpha_4\alpha_7+3\alpha_3^2\alpha_4\alpha_7+$
$4\alpha_0\alpha_4^2\alpha_7 + 3\alpha_1\alpha_4^2\alpha_7 + 2\alpha_2\alpha_4^2\alpha_7 + \alpha_3\alpha_4^2\ \alpha_7 + 161\alpha_0^2\alpha_6\alpha_7 + 200\alpha_0\alpha_1\alpha_6\alpha_7 + 78\alpha_1^2\alpha_6\alpha_7 + 150\alpha_0\alpha_2\alpha_6\alpha_7 +$
$117\alpha_1\alpha_2\alpha_6\alpha_7 + 42\alpha_2^2\alpha_6\alpha_7 + 100\alpha_0\alpha_3\alpha_6\alpha_7 + 78\alpha_1\alpha_3\alpha_6\alpha_7 + 56\alpha_2\alpha_3\alpha_6\alpha_7 + 17\alpha_3^2\alpha_6\alpha_7 + 50\alpha_0\alpha_4\alpha_6\alpha_7 +$
$39\alpha_1\alpha_4\alpha_6\alpha_7 + 28\alpha_2\alpha_4\alpha_6\alpha_7 + 17\alpha_3\alpha_4\alpha_6\alpha_7 + 3\alpha_4^2\alpha_6\alpha_7 + 108\alpha_0\alpha_6^2\alpha_7 + 72\alpha_1\alpha_6^2\alpha_7 + 54\alpha_2\alpha_6^2\alpha_7 + 36\alpha_3\alpha_6^2\alpha_7 +$
$18\alpha_4\alpha_6^2\alpha_7 + 24\alpha_6^3\alpha_7 + 41\alpha_0^2\alpha_7^2 + 56\alpha_0\alpha_1\alpha_7^2 + 22\alpha_1^2\alpha_7^2 + 42\alpha_0\alpha_2\alpha_7^2 + 33\alpha_1\alpha_2\alpha_7^2 + 12\alpha_2^2\alpha_7^2 + 28\alpha_0\alpha_3\alpha_7^2 +$
$22\alpha_1\alpha_3\alpha_7^2 + 16\alpha_2\alpha_3\alpha_7^2 + 5\alpha_3^2\alpha_7^2 + 14\alpha_0\alpha_4\alpha_7^2 + 11\alpha_1\alpha_4\alpha_7^2 + 8\alpha_2\alpha_4\alpha_7^2 + 5\alpha_3\alpha_4\alpha_7^2 + \alpha_4^2\alpha_7^2 + 56\alpha_0\alpha_6\alpha_7^2 +$
$40\alpha_1\alpha_6\alpha_7^2+30\alpha_2\alpha_6\alpha_7^2+20\alpha_3\alpha_6\alpha_7^2+10\alpha_4\alpha_6\alpha_7^2+19\alpha_6^2\alpha_7^2+10\alpha_0\alpha_7^3+8\alpha_1\alpha_7^3+6\alpha_2\alpha_7^3+4\alpha_3\alpha_7^3+2\alpha_4\alpha_7^3+7\alpha_6\alpha_7^3+$
$\alpha_7^4 + 120\alpha_0^3\alpha_8 + 216\alpha_0^2\alpha_1\alpha_8 + 60\alpha_0\alpha_1^2\alpha_8 + 24\alpha_1^3\alpha_8 + 162\alpha_0^2\alpha_2\alpha_8 + 90\alpha_0\alpha_1\alpha_2\alpha_8 + 54\alpha_1^2\alpha_2\alpha_8 + 36\alpha_0\alpha_2^2\alpha_8 +$
$36\alpha_1\alpha_2^2\alpha_8 + 6\alpha_2^3\alpha_8 + 108\alpha_0^2\alpha_3\alpha_8 + 60\alpha_0\alpha_1\alpha_3\alpha_8 + 36\alpha_1^2\alpha_3\alpha_8 + 48\alpha_0\alpha_2\alpha_3\alpha_8 + 48\alpha_1\alpha_2\alpha_3\alpha_8 + 12\alpha_2^2\alpha_3\alpha_8 +$
$18\alpha_0\alpha_3^2\alpha_8 + 12\alpha_1\alpha_3^2\alpha_8 + 6\alpha_2\alpha_3^2\alpha_8 + 54\alpha_0^2\alpha_4\alpha_8 + 30\alpha_0\alpha_1\alpha_4\alpha_8 + 18\alpha_1^2\alpha_4\alpha_8 + 24\alpha_0\alpha_2\alpha_4\alpha_8 + 24\alpha_1\alpha_2\alpha_4\alpha_8 +$
$6\alpha_2^2\alpha_4\alpha_8 + 18\alpha_0\alpha_3\alpha_4\alpha_8 + 12\alpha_1\alpha_3\alpha_4\alpha_8 + 6\alpha_2\alpha_3\alpha_4\alpha_8 + 6\alpha_0\alpha_4^2\alpha_8 + 240\alpha_0^2\alpha_6\alpha_8 + 288\alpha_0\alpha_1\alpha_6\alpha_8 + 76\alpha_1^2\alpha_6\alpha_8 +$
$216\alpha_0\alpha_2\alpha_6\alpha_8 + 114\alpha_1\alpha_2\alpha_6\alpha_8 + 42\alpha_2^2\alpha_6\alpha_8 + 144\alpha_0\alpha_3\alpha_6\alpha_8 + 76\alpha_1\alpha_3\alpha_6\alpha_8 + 56\alpha_2\alpha_3\alpha_6\alpha_8 + 18\alpha_3^2\alpha_6\alpha_8 +$
$72\alpha_0\alpha_4\alpha_6\alpha_8 + 38\alpha_1\alpha_4\alpha_6\alpha_8 + 28\alpha_2\alpha_4\alpha_6\alpha_8 + 18\alpha_3\alpha_4\alpha_6\alpha_8 + 4\alpha_4^2\alpha_6\alpha_8 + 161\alpha_0\alpha_6^2\alpha_8 + 100\alpha_1\alpha_6^2\alpha_8 + 75\alpha_2\alpha_6^2\alpha_8 +$
$50\alpha_3\alpha_6^2\alpha_8 + 25\alpha_4\alpha_6^2\alpha_8 + 36\alpha_6^3\alpha_8 + 120\alpha_0^2\alpha_7\alpha_8 + 144\alpha_0\alpha_1\alpha_7\alpha_8 + 38\alpha_1^2\alpha_7\alpha_8 + 108\alpha_0\alpha_2\alpha_7\alpha_8 + 57\alpha_1\alpha_2\alpha_7\alpha_8 +$
$21\alpha_2^2\alpha_7\alpha_8 + 72\alpha_0\alpha_3\alpha_7\alpha_8 + 38\alpha_1\alpha_3\alpha_7\alpha_8 + 28\alpha_2\alpha_3\alpha_7\alpha_8 + 9\alpha_3^2\alpha_7\alpha_8 + 36\alpha_0\alpha_4\alpha_7\alpha_8 + 19\alpha_1\alpha_4\alpha_7\alpha_8 + 14\alpha_2\alpha_4\alpha_7\alpha_8 +$
$9\alpha_3\alpha_4\alpha_7\alpha_8 + 2\alpha_4^2\alpha_7\alpha_8 + 161\alpha_0\alpha_6\alpha_7\alpha_8 + 100\alpha_1\alpha_6\alpha_7\alpha_8 + 75\alpha_2\alpha_6\alpha_7\alpha_8 + 50\alpha_3\alpha_6\alpha_7\alpha_8 + 25\alpha_4\alpha_6\alpha_7\alpha_8 +$
$54\alpha_6^2\alpha_7\alpha_8 + 41\alpha_0\alpha_7^2\alpha_8 + 28\alpha_1\alpha_7^2\alpha_8 + 21\alpha_2\alpha_7^2\alpha_8 + 14\alpha_3\alpha_7^2\alpha_8 + 7\alpha_4\alpha_7^2\alpha_8 + 28\alpha_6\alpha_7^2\alpha_8 + 5\alpha_7^3\alpha_8 + 87\alpha_0^2\alpha_8^2 +$
$96\alpha_0\alpha_1\alpha_8^2 + 10\alpha_1^2\alpha_8^2 + 72\alpha_0\alpha_2\alpha_8^2 + 15\alpha_1\alpha_2\alpha_8^2 + 6\alpha_2^2\alpha_8^2 + 48\alpha_0\alpha_3\alpha_8^2 + 10\alpha_1\alpha_3\alpha_8^2 + 8\alpha_2\alpha_3\alpha_8^2 + 3\alpha_3^2\alpha_8^2 +$
$24\alpha_0\alpha_4\alpha_8^2 + 5\alpha_1\alpha_4\alpha_8^2 + 4\alpha_2\alpha_4\alpha_8^2 + 3\alpha_3\alpha_4\alpha_8^2 + \alpha_4^2\alpha_8^2 + 116\alpha_0\alpha_6\alpha_8^2 + 64\alpha_1\alpha_6\alpha_8^2 + 48\alpha_2\alpha_6\alpha_8^2 + 32\alpha_3\alpha_6\alpha_8^2 +$
$16\alpha_4\alpha_6\alpha_8^2 + 39\alpha_6^2\alpha_8^2 + 58\alpha_0\alpha_7\alpha_8^2 + 32\alpha_1\alpha_7\alpha_8^2 + 24\alpha_2\alpha_7\alpha_8^2 + 16\alpha_3\alpha_7\alpha_8^2 + 8\alpha_4\alpha_7\alpha_8^2 + 39\alpha_6\alpha_7\alpha_8^2 + 10\alpha_7^2\alpha_8^2 +$
$27\alpha_0\alpha_8^3 + 12\alpha_1\alpha_8^3 + 9\alpha_2\alpha_8^3 + 6\alpha_3\alpha_8^3 + 3\alpha_4\alpha_8^3 + 18\alpha_6\alpha_8^3 + 9\alpha_7\alpha_8^3 + 3\alpha_8^4)) + p(-\alpha_1(\alpha_1+\alpha_2)(\alpha_1+\alpha_2+\ \alpha_3)(\alpha_1+$
$\alpha_2+\alpha_3+\alpha_4)(\alpha_1\ +\alpha_2+\alpha_3+\alpha_4+\alpha_5) + 3q^4\alpha_0^2(\alpha_0+\alpha_8)^2(2\alpha_0+\ \alpha_8) + 2q^3\alpha_0(\alpha_0+\alpha_8)(18\alpha_0^3+48\alpha_0^2\alpha_1+20\alpha_0\alpha_1^2+$
$36\alpha_0^2\alpha_2 + 30\alpha_0\alpha_1\alpha_2 + 12\alpha_0\alpha_2^2 + 24\alpha_0^2\alpha_3 + 20\alpha_0\alpha_1\alpha_3 + 16\alpha_0\alpha_2\alpha_3 + 6\alpha_0\alpha_3^2 + 12\alpha_0^2\alpha_4 + 10\alpha_0\alpha_1\alpha_4 + 8\alpha_0\alpha_2\alpha_4 +$
$6\alpha_0\alpha_3\alpha_4 + 2\alpha_0\alpha_4^2 + 30\alpha_0^2\alpha_6 + 32\alpha_0\alpha_1\alpha_6 + 24\alpha_0\alpha_2\alpha_6 + 16\alpha_0\alpha_3\alpha_6 + 8\alpha_0\alpha_4\alpha_6 + 12\alpha_0\alpha_6^2 + 15\alpha_0^2\alpha_7 + 16\alpha_0\alpha_1\alpha_7 +$
$12\alpha_0\alpha_2\alpha_7 + 8\alpha_0\alpha_3\alpha_7 + 4\alpha_0\alpha_4\alpha_7 + 12\alpha_0\alpha_6\alpha_7 + 2\alpha_0\alpha_7^2 + 27\alpha_0^2\alpha_8 + 48\alpha_0\alpha_1\alpha_8 + 10\alpha_1^2\alpha_8 + 36\alpha_0\alpha_2\alpha_8 +$
$15\alpha_1\alpha_2\alpha_8 + 6\alpha_2^2\alpha_8 + 24\alpha_0\alpha_3\alpha_8 + 10\alpha_1\alpha_3\alpha_8 + 8\alpha_2\alpha_3\alpha_8 + 3\alpha_3^2\alpha_8 + 12\alpha_0\alpha_4\alpha_8 + 5\alpha_1\alpha_4\alpha_8 + 4\alpha_2\alpha_4\alpha_8 + 3\alpha_3\alpha_4\alpha_8 +$
$\alpha_4^2\alpha_8 + 30\alpha_0\alpha_6\alpha_8 + 16\alpha_1\alpha_6\alpha_8 + 12\alpha_2\alpha_6\alpha_8 + 8\alpha_3\alpha_6\alpha_8 + 4\alpha_4\alpha_6\alpha_8 + 6\alpha_6^2\alpha_8 + 15\alpha_0\alpha_7\alpha_8 + 8\alpha_1\alpha_7\alpha_8 + 6\alpha_2\alpha_7\alpha_8 +$
$4\alpha_3\alpha_7\alpha_8 + 2\alpha_4\alpha_7\alpha_8 + 6\alpha_6\alpha_7\alpha_8 + \alpha_7^2\alpha_8 + 15\alpha_0\alpha_8^2 + 12\alpha_1\alpha_8^2 + 9\alpha_2\alpha_8^2 + 6\alpha_3\alpha_8^2 + 3\alpha_4\alpha_8^2 + 8\alpha_6\alpha_8^2 + 4\alpha_7\alpha_8^2 + 3\alpha_8^3) +$
$q^2(-114\alpha_0^5 - 288\alpha_0^4\alpha_1 - 228\alpha_0^3\alpha_1^2 - 108\alpha_0^2\alpha_1^3 - 30\alpha_0\alpha_1^4 - 216\alpha_0^4\alpha_2 - 342\alpha_0^3\alpha_1\alpha_2 - 243\alpha_0^2\alpha_1^2\alpha_2 - 90\alpha_0\alpha_1^3\alpha_2 -$
$126\alpha_0^3\alpha_2^2 - 171\alpha_0^2\alpha_1\alpha_2^2 - 96\alpha_0\alpha_1^2\alpha_2^2 - 36\alpha_0^2\alpha_2^3 - 42\alpha_0\alpha_1\alpha_2^3 - 6\alpha_0\alpha_2^4 - 144\alpha_0^4\alpha_3 - 228\alpha_0^3\alpha_1\alpha_3 - 162\alpha_0^2\alpha_1^2\alpha_3 -$
$60\alpha_0\alpha_1^3\alpha_3 - 168\alpha_0^3\alpha_2\alpha_3 - 228\alpha_0^2\alpha_1\alpha_2\alpha_3 - 128\alpha_0\alpha_1^2\alpha_2\alpha_3 - 72\alpha_0^2\alpha_2^2\alpha_3 - 84\alpha_0\alpha_1\alpha_2^2\alpha_3 - 16\alpha_0\alpha_2^3\alpha_3 - 54\alpha_0^3\alpha_3^2 -$
$66\alpha_0^2\alpha_1\alpha_3^2 - 38\alpha_0\alpha_1^2\alpha_3^2 - 42\alpha_0^2\alpha_2\alpha_3^2 - 50\alpha_0\alpha_1\alpha_2\alpha_3^2 - 14\alpha_0\alpha_2^2\alpha_3^2 - 6\alpha_0^2\alpha_3^3 - 8\alpha_0\alpha_1\alpha_3^3 - 4\alpha_0\alpha_2\alpha_3^3 - 72\alpha_0^4\alpha_4 -$
$114\alpha_0^3\alpha_1\alpha_4 - 81\alpha_0^2\alpha_1^2\alpha_4 - 30\alpha_0\alpha_1^3\alpha_4 - 84\alpha_0^3\alpha_2\alpha_4 - 114\alpha_0^2\alpha_1\alpha_2\alpha_4 - 64\alpha_0\alpha_1^2\alpha_2\alpha_4 - 36\alpha_0^2\alpha_2^2\alpha_4 - 42\alpha_0\alpha_1\alpha_2^2\alpha_4 -$
$8\alpha_0\alpha_2^3\alpha_4 - 54\alpha_0^3\alpha_3\alpha_4 - 66\alpha_0^2\alpha_1\alpha_3\alpha_4 - 38\alpha_0\alpha_1^2\alpha_3\alpha_4 - 42\alpha_0^2\alpha_2\alpha_3\alpha_4 - 50\alpha_0\alpha_1\alpha_2\alpha_3\alpha_4 - 14\alpha_0\alpha_2^2\alpha_3\alpha_4 -$
$9\alpha_0^2\alpha_3^2\alpha_4 - 12\alpha_0\alpha_1\alpha_3^2\alpha_4 - 6\alpha_0\alpha_2\alpha_3^2\alpha_4 - 12\alpha_0^3\alpha_4^2 - 9\alpha_0^2\alpha_1\alpha_4^2 - 6\alpha_0\alpha_1^2\alpha_4^2 - 6\alpha_0^2\alpha_2\alpha_4^2 - 8\alpha_0\alpha_1\alpha_2\alpha_4^2 - 2\alpha_0\alpha_2^2\alpha_4^2 -$
$3\alpha_0^2\alpha_3\alpha_4^2 - 4\alpha_0\alpha_1\alpha_3\alpha_4^2 - 2\alpha_0\alpha_2\alpha_3\alpha_4^2 - 300\alpha_0^4\alpha_6 - 576\alpha_0^3\alpha_1\alpha_6 - 376\alpha_0^2\alpha_1^2\alpha_6 - 112\alpha_0\alpha_1^3\alpha_6 - 432\alpha_0^3\alpha_2\alpha_6 -$
$564\alpha_0^2\alpha_1\alpha_2\alpha_6 - 252\alpha_0\alpha_1^2\alpha_2\alpha_6 - 204\alpha_0^2\alpha_2^2\alpha_6 - 180\alpha_0\alpha_1\alpha_2^2\alpha_6 - 40\alpha_0\alpha_2^3\alpha_6 - 288\alpha_0^3\alpha_3\alpha_6 - 376\alpha_0^2\alpha_1\alpha_3\alpha_6 -$
$168\alpha_0\alpha_1^2\alpha_3\alpha_6 - 272\alpha_0^2\alpha_2\alpha_3\alpha_6 - 240\alpha_0\alpha_1\alpha_2\alpha_3\alpha_6 - 80\alpha_0\alpha_2^2\alpha_3\alpha_6 - 84\alpha_0^2\alpha_3^2\alpha_6 - 72\alpha_0\alpha_1\alpha_3^2\alpha_6 - 48\alpha_0\alpha_2\alpha_3^2\alpha_6 -$
$8\alpha_0\alpha_3^3\alpha_6 - 144\alpha_0^3\alpha_4\alpha_6 - 188\alpha_0^2\alpha_1\alpha_4\alpha_6 - 84\alpha_0\alpha_1^2\alpha_4\alpha_6 - 136\alpha_0^2\alpha_2\alpha_4\alpha_6 - 120\alpha_0\alpha_1\alpha_2\alpha_4\alpha_6 - 40\alpha_0\alpha_2^2\alpha_4\alpha_6 -$
$84\alpha_0^2\alpha_3\alpha_4\alpha_6 - 72\alpha_0\alpha_1\alpha_3\alpha_4\alpha_6 - 48\alpha_0\alpha_2\alpha_3\alpha_4\alpha_6 - 12\alpha_0\alpha_3^2\alpha_4\alpha_6 - 16\alpha_0^2\alpha_4^2\alpha_6 - 12\alpha_0\alpha_1\alpha_4^2\alpha_6 - 8\alpha_0\alpha_2\alpha_4^2\alpha_6 -$
$4\alpha_0\alpha_3\alpha_4^2\alpha_6 - 298\alpha_0^3\alpha_6^2 - 400\alpha_0^2\alpha_1\alpha_6^2 - 156\alpha_0\alpha_1^2\alpha_6^2 - 300\alpha_0^2\alpha_2\alpha_6^2 - 234\alpha_0\alpha_1\alpha_2\alpha_6^2 - 84\alpha_0\alpha_2^2\alpha_6^2 - 200\alpha_0^2\alpha_3\alpha_6^2 -$
$156\alpha_0\alpha_1\alpha_3\alpha_6^2 - 112\alpha_0\alpha_2\alpha_3\alpha_6^2 - 34\alpha_0\alpha_3^2\alpha_6^2 - 100\alpha_0^2\alpha_4\alpha_6^2 - 78\alpha_0\alpha_1\alpha_4\alpha_6^2 - 56\alpha_0\alpha_2\alpha_4\alpha_6^2 - 34\alpha_0\alpha_3\alpha_4\alpha_6^2 -$
$6\alpha_0\alpha_4^2\alpha_6^2 - 132\alpha_0^2\alpha_6^3 - 96\alpha_0\alpha_1\alpha_6^3 - 72\alpha_0\alpha_2\alpha_6^3 - 48\alpha_0\alpha_3\alpha_6^3 - 24\alpha_0\alpha_4\alpha_6^3 - 22\alpha_0\alpha_6^4 - 150\alpha_0^4\alpha_7 - 288\alpha_0^3\alpha_1\alpha_7 -$
$188\alpha_0^2\alpha_1^2\alpha_7 - 56\alpha_0\alpha_1^3\alpha_7 - 216\alpha_0^3\alpha_2\alpha_7 - 282\alpha_0^2\alpha_1\alpha_2\alpha_7 - 126\alpha_0\alpha_1^2\alpha_2\alpha_7 - 102\alpha_0^2\alpha_2^2\alpha_7 - 90\alpha_0\alpha_1\alpha_2^2\alpha_7 -$
$20\alpha_0\alpha_2^3\alpha_7 - 144\alpha_0^3\alpha_3\alpha_7 - 188\alpha_0^2\alpha_1\alpha_3\alpha_7 - 84\alpha_0\alpha_1^2\alpha_3\alpha_7 - 136\alpha_0^2\alpha_2\alpha_3\alpha_7 - 120\alpha_0\alpha_1\alpha_2\alpha_3\alpha_7 - 40\alpha_0\alpha_2^2\alpha_3\alpha_7 -$
$42\alpha_0^2\alpha_3^2\alpha_7 - 36\alpha_0\alpha_1\alpha_3^2\alpha_7 - 24\alpha_0\alpha_2\alpha_3^2\alpha_7 - 4\alpha_0\alpha_3^3\alpha_7 - 72\alpha_0^3\alpha_4\alpha_7 - 94\alpha_0^2\alpha_1\alpha_4\alpha_7 - 42\alpha_0\alpha_1^2\alpha_4\alpha_7 - 68\alpha_0^2\alpha_2\alpha_4\alpha_7 -$
$60\alpha_0\alpha_1\alpha_2\alpha_4\alpha_7 - 20\alpha_0\alpha_2^2\alpha_4\alpha_7 - 42\alpha_0^2\alpha_3\alpha_4\alpha_7 - 36\alpha_0\alpha_1\alpha_3\alpha_4\alpha_7 - 24\alpha_0\alpha_2\alpha_3\alpha_4\alpha_7 - 6\alpha_0\alpha_3^2\alpha_4\alpha_7 - 8\alpha_0^2\alpha_4^2\alpha_7 -$
$6\alpha_0\alpha_1\alpha_4^2\alpha_7 - 4\alpha_0\alpha_2\alpha_4^2\alpha_7 - 2\alpha_0\alpha_3\alpha_4^2\alpha_7 - 298\alpha_0^3\alpha_6\alpha_7 - 400\alpha_0^2\alpha_1\alpha_6\alpha_7 - 156\alpha_0\alpha_1^2\alpha_6\alpha_7 - 300\alpha_0^2\alpha_2\alpha_6\alpha_7 -$
$234\alpha_0\alpha_1\alpha_2\alpha_6\alpha_7 - 84\alpha_0\alpha_2^2\alpha_6\alpha_7 - 200\alpha_0^2\alpha_3\alpha_6\alpha_7 - 156\alpha_0\alpha_1\alpha_3\alpha_6\alpha_7 - 112\alpha_0\alpha_2\alpha_3\alpha_6\alpha_7 - 34\alpha_0\alpha_3^2\alpha_6\alpha_7 -$
$100\alpha_0^2\alpha_4\alpha_6\alpha_7 - 78\alpha_0\alpha_1\alpha_4\alpha_6\alpha_7 - 56\alpha_0\alpha_2\alpha_4\alpha_6\alpha_7 - 34\alpha_0\alpha_3\alpha_4\alpha_6\alpha_7 - 6\alpha_0\alpha_4^2\alpha_6\alpha_7 - 198\alpha_0^2\alpha_6^2\alpha_7 - 144\alpha_0\alpha_1\alpha_6^2\alpha_7 -$
$108\alpha_0\alpha_2\alpha_6^2\alpha_7 - 72\alpha_0\alpha_3\alpha_6^2\alpha_7 - 36\alpha_0\alpha_4\alpha_6^2\alpha_7 - 44\alpha_0\alpha_6^3\alpha_7 - 78\alpha_0^3\alpha_7^2 - 112\alpha_0^2\alpha_1\alpha_7^2 - 44\alpha_0\alpha_1^2\alpha_7^2 - 84\alpha_0^2\alpha_2\alpha_7^2 -$

$66\alpha_0\alpha_1\alpha_2\alpha_7^2-24\alpha_0\alpha_2^2\alpha_7^2-56\alpha_0^2\alpha_3\alpha_7^2-44\alpha_0\alpha_1\alpha_3\alpha_7^2-32\alpha_0\alpha_2\alpha_3\alpha_7^2-10\alpha_0\alpha_3^2\alpha_7^2-28\alpha_0^2\alpha_4\alpha_7^2-22\alpha_0\alpha_1\alpha_4\alpha_7^2-$
$16\alpha_0\alpha_2\alpha_4\alpha_7^2-10\alpha_0\alpha_3\alpha_4\alpha_7^2-2\alpha_0\alpha_4^2\alpha_7^2-106\alpha_0^2\alpha_6\alpha_7^2-80\alpha_0\alpha_1\alpha_6\alpha_7^2-60\alpha_0\alpha_2\alpha_6\alpha_7^2-40\alpha_0\alpha_3\alpha_6\alpha_7^2-20\alpha_0\alpha_4\alpha_6\alpha_7^2-$
$36\alpha_0\alpha_6^2\alpha_7^2-20\alpha_0^2\alpha_7^3-16\alpha_0\alpha_1\alpha_7^3-12\alpha_0\alpha_2\alpha_7^3-8\alpha_0\alpha_3\alpha_7^3-4\alpha_0\alpha_4\alpha_7^3-14\alpha_0\alpha_6\alpha_7^3-2\alpha_0\alpha_7^4-285\alpha_0^4\alpha_8-$
$576\alpha_0^3\alpha_1\alpha_8-342\alpha_0^2\alpha_1^2\alpha_8-108\alpha_0\alpha_1^3\alpha_8-15\alpha_1^4\alpha_8-432\alpha_0^3\alpha_2\alpha_8-513\alpha_0^2\alpha_1\alpha_2\alpha_8-243\alpha_0\alpha_1^2\alpha_2\alpha_8-45\alpha_1^3\alpha_2\alpha_8-$
$189\alpha_0^2\alpha_2^2\alpha_8-171\alpha_0\alpha_1\alpha_2^2\alpha_8-48\alpha_1^2\alpha_2^2\alpha_8-36\alpha_0\alpha_2^3\alpha_8-21\alpha_1\alpha_2^3\alpha_8-3\alpha_2^4\alpha_8-288\alpha_0^3\alpha_3\alpha_8-342\alpha_0^2\alpha_1\alpha_3\alpha_8-$
$162\alpha_0\alpha_1^2\alpha_3\alpha_8-30\alpha_1^3\alpha_3\alpha_8-252\alpha_0^2\alpha_2\alpha_3\alpha_8-228\alpha_0\alpha_1\alpha_2\alpha_3\alpha_8-64\alpha_1^2\alpha_2\alpha_3\alpha_8-72\alpha_0\alpha_2^2\alpha_3\alpha_8-42\alpha_1\alpha_2^2\alpha_3\alpha_8-$
$8\alpha_2^3\alpha_3\alpha_8-81\alpha_0^2\alpha_3^2\alpha_8-66\alpha_0\alpha_1\alpha_3^2\alpha_8-19\alpha_1^2\alpha_3^2\alpha_8-42\alpha_0\alpha_2\alpha_3^2\alpha_8-25\alpha_1\alpha_2\alpha_3^2\alpha_8-7\alpha_2^2\alpha_3^2\alpha_8-6\alpha_0\alpha_3^3\alpha_8-$
$4\alpha_1\alpha_3^3\alpha_8-2\alpha_2\alpha_3^3\alpha_8-144\alpha_0^3\alpha_4\alpha_8-171\alpha_0^2\alpha_1\alpha_4\alpha_8-81\alpha_0\alpha_1^2\alpha_4\alpha_8-15\alpha_1^3\alpha_4\alpha_8-126\alpha_0^2\alpha_2\alpha_4\alpha_8-114\alpha_0\alpha_1\alpha_2\alpha_4\alpha_8-$
$32\alpha_1^2\alpha_2\alpha_4\alpha_8-36\alpha_0\alpha_2^2\alpha_4\alpha_8-21\alpha_1\alpha_2^2\alpha_4\alpha_8-4\alpha_2^3\alpha_4\alpha_8-81\alpha_0^2\alpha_3\alpha_4\alpha_8-66\alpha_0\alpha_1\alpha_3\alpha_4\alpha_8-19\alpha_1^2\alpha_3\alpha_4\alpha_8-$
$42\alpha_0\alpha_2\alpha_3\alpha_4\alpha_8-25\alpha_1\alpha_2\alpha_3\alpha_4\alpha_8-7\alpha_2^2\alpha_3\alpha_4\alpha_8-9\alpha_0\alpha_3^2\alpha_4\alpha_8-6\alpha_1\alpha_3^2\alpha_4\alpha_8-3\alpha_2\alpha_3^2\alpha_4\alpha_8-18\alpha_0^2\alpha_4^2\alpha_8-$
$9\alpha_0\alpha_1\alpha_4^2\alpha_8-3\alpha_1^2\alpha_4^2\alpha_8-6\alpha_0\alpha_2\alpha_4^2\alpha_8-4\alpha_1\alpha_2\alpha_4^2\alpha_8-\alpha_2^2\,\alpha_4^2\alpha_8-3\alpha_0\alpha_3\alpha_4^2\alpha_8-2\alpha_1\alpha_3\alpha_4^2\alpha_8-\alpha_2\,\alpha_3\alpha_4^2\alpha_8-$
$600\alpha_0^3\alpha_6\alpha_8-864\alpha_0^2\alpha_1\alpha_6\alpha_8-376\alpha_0\alpha_1^2\alpha_6\alpha_8-56\alpha_1^3\alpha_6\alpha_8-648\alpha_0^2\alpha_2\alpha_6\alpha_8-564\alpha_0\alpha_1\alpha_2\alpha_6\alpha_8-126\alpha_1^2\alpha_2\alpha_6\alpha_8-$
$204\alpha_0\alpha_2^2\alpha_6\alpha_8-90\alpha_1\alpha_2^2\alpha_6\alpha_8-20\alpha_2^3\alpha_6\alpha_8-432\alpha_0^2\alpha_3\alpha_6\alpha_8-376\alpha_0\alpha_1\alpha_3\alpha_6\alpha_8-84\alpha_1^2\alpha_3\alpha_6\alpha_8-272\alpha_0\alpha_2\alpha_3\alpha_6\alpha_8-$
$120\alpha_1\alpha_2\alpha_3\alpha_6\alpha_8-40\alpha_2^2\alpha_3\alpha_6\alpha_8-84\alpha_0\alpha_3^2\alpha_6\alpha_8-36\alpha_1\alpha_3^2\alpha_6\alpha_8-24\alpha_2\alpha_3^2\alpha_6\alpha_8-4\alpha_3^3\alpha_6\alpha_8-216\alpha_0^2\alpha_4\alpha_6\alpha_8-$
$188\alpha_0\alpha_1\alpha_4\alpha_6\alpha_8-42\alpha_1^2\alpha_4\alpha_6\alpha_8-136\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8-60\alpha_1\alpha_2\alpha_4\alpha_6\alpha_8-20\alpha_2^2\alpha_4\alpha_6\alpha_8-84\alpha_0\alpha_3\alpha_4\alpha_6\alpha_8-$
$36\alpha_1\alpha_3\alpha_4\alpha_6\alpha_8-24\alpha_2\alpha_3\alpha_4\alpha_6\alpha_8-6\alpha_3^2\alpha_4\alpha_6\alpha_8-16\alpha_0\alpha_4^2\alpha_6\alpha_8-6\alpha_1\alpha_4^2\alpha_6\alpha_8-4\alpha_2\alpha_4^2\alpha_6\alpha_8-2\alpha_3\alpha_4^2\alpha_6\alpha_8-$
$447\alpha_0^2\alpha_6^2\alpha_8-400\alpha_0\alpha_1\alpha_6^2\alpha_8-78\alpha_1^2\alpha_6^2\alpha_8-300\alpha_0\alpha_2\alpha_6^2\alpha_8-117\alpha_1\alpha_2\alpha_6^2\alpha_8-42\alpha_2^2\alpha_6^2\alpha_8-200\alpha_0\alpha_3\alpha_6^2\alpha_8-$
$78\alpha_1\alpha_3\alpha_6^2\alpha_8-56\alpha_2\alpha_3\alpha_6^2\alpha_8-17\alpha_3^2\alpha_6^2\alpha_8-100\alpha_0\alpha_4\alpha_6^2\alpha_8-39\alpha_1\alpha_4\alpha_6^2\alpha_8-28\alpha_2\alpha_4\alpha_6^2\alpha_8-17\alpha_3\alpha_4\alpha_6^2\alpha_8-$
$3\alpha_4^2\alpha_6^2\alpha_8-132\alpha_0\alpha_6^3\alpha_8-48\alpha_1\alpha_6^3\alpha_8-36\alpha_2\alpha_6^3\alpha_8-24\alpha_3\alpha_6^3\alpha_8-12\alpha_4\alpha_6^3\alpha_8-11\alpha_6^4\alpha_8-300\alpha_0^3\alpha_7\alpha_8-432\alpha_0^2\alpha_1\alpha_7\alpha_8-$
$188\alpha_0\alpha_1^2\alpha_7\alpha_8-28\alpha_1^3\alpha_7\alpha_8-324\alpha_0^2\alpha_2\alpha_7\alpha_8-282\alpha_0\alpha_1\alpha_2\alpha_7\alpha_8-63\alpha_1^2\alpha_2\alpha_7\alpha_8-102\alpha_0\alpha_2^2\alpha_7\alpha_8-45\alpha_1\alpha_2^2\alpha_7\alpha_8-$
$10\alpha_2^3\alpha_7\alpha_8-216\alpha_0^2\alpha_3\alpha_7\alpha_8-188\alpha_0\alpha_1\alpha_3\alpha_7\alpha_8-42\alpha_1^2\alpha_3\alpha_7\alpha_8-136\alpha_0\alpha_2\alpha_3\alpha_7\alpha_8-60\alpha_1\alpha_2\alpha_3\alpha_7\alpha_8-20\alpha_2^2\alpha_3\alpha_7\alpha_8-$
$42\alpha_0\alpha_3^2\alpha_7\alpha_8-18\alpha_1\alpha_3^2\alpha_7\alpha_8-12\alpha_2\alpha_3^2\alpha_7\alpha_8-2\alpha_3^3\alpha_7\alpha_8-108\alpha_0^2\alpha_4\alpha_7\alpha_8-94\alpha_0\alpha_1\alpha_4\alpha_7\alpha_8-21\alpha_1^2\alpha_4\alpha_7\alpha_8-$
$68\alpha_0\alpha_2\alpha_4\alpha_7\alpha_8-30\alpha_1\alpha_2\alpha_4\alpha_7\alpha_8-10\alpha_2^2\alpha_4\alpha_7\alpha_8-42\alpha_0\alpha_3\alpha_4\alpha_7\alpha_8-18\alpha_1\alpha_3\alpha_4\alpha_7\alpha_8-12\alpha_2\alpha_3\alpha_4\alpha_7\alpha_8-$
$3\alpha_3^2\alpha_4\alpha_7\alpha_8-8\alpha_0\alpha_4^2\alpha_7\alpha_8-3\alpha_1\alpha_4^2\alpha_7\alpha_8-2\alpha_2\alpha_4^2\alpha_7\alpha_8-\alpha_3\,\alpha_4^2\alpha_7\alpha_8-447\alpha_0^2\alpha_6\alpha_7\alpha_8-400\alpha_0\alpha_1\alpha_6\alpha_7\alpha_8-$
$78\alpha_1^2\alpha_6\alpha_7\alpha_8-300\alpha_0\alpha_2\alpha_6\alpha_7\alpha_8-117\alpha_1\alpha_2\alpha_6\alpha_7\alpha_8-42\alpha_2^2\alpha_6\alpha_7\alpha_8-200\alpha_0\alpha_3\alpha_6\alpha_7\alpha_8-78\alpha_1\alpha_3\alpha_6\alpha_7\alpha_8-$
$56\alpha_2\alpha_3\alpha_6\alpha_7\alpha_8-17\alpha_3^2\alpha_6\alpha_7\alpha_8-100\alpha_0\alpha_4\alpha_6\alpha_7\alpha_8-39\alpha_1\alpha_4\alpha_6\alpha_7\alpha_8-28\alpha_2\alpha_4\alpha_6\alpha_7\alpha_8-17\alpha_3\alpha_4\alpha_6\alpha_7\alpha_8-$
$3\alpha_4^2\alpha_6\alpha_7\alpha_8-198\alpha_0\alpha_6^2\alpha_7\alpha_8-72\alpha_1\alpha_6^2\alpha_7\alpha_8-54\alpha_2\alpha_6^2\alpha_7\alpha_8-36\alpha_3\alpha_6^2\alpha_7\alpha_8-18\alpha_4\alpha_6^2\alpha_7\alpha_8-22\alpha_6^3\alpha_7\alpha_8-$
$117\alpha_0^2\alpha_7^2\alpha_8-112\alpha_0\alpha_1\alpha_7^2\alpha_8-22\alpha_1^2\alpha_7^2\alpha_8-84\alpha_0\alpha_2\alpha_7^2\alpha_8-33\alpha_1\alpha_2\alpha_7^2\alpha_8-12\alpha_2^2\alpha_7^2\alpha_8-56\alpha_0\alpha_3\alpha_7^2\alpha_8-22\alpha_1\alpha_3\alpha_7^2\alpha_8-$
$16\alpha_2\alpha_3\alpha_7^2\alpha_8-5\alpha_3^2\alpha_7^2\alpha_8-28\alpha_0\alpha_4\alpha_7^2\alpha_8-11\alpha_1\alpha_4\alpha_7^2\alpha_8-8\alpha_2\alpha_4\alpha_7^2\alpha_8-5\alpha_3\alpha_4\alpha_7^2\alpha_8-\alpha_4^2\,\alpha_7^2\alpha_8-106\alpha_0\alpha_6\alpha_7^2\alpha_8-$
$40\alpha_1\alpha_6\alpha_7^2\alpha_8-30\alpha_2\alpha_6\alpha_7^2\alpha_8-20\alpha_3\alpha_6\alpha_7^2\alpha_8-10\alpha_4\alpha_6\alpha_7^2\alpha_8-18\alpha_6^2\alpha_7^2\alpha_8-20\alpha_0\alpha_7^3\alpha_8-8\alpha_1\alpha_7^3\alpha_8-6\alpha_2\alpha_7^3\alpha_8-$
$4\alpha_3\alpha_7^3\alpha_8-2\alpha_4\alpha_7^3\alpha_8-7\alpha_6\alpha_7^3\alpha_8-\alpha_7^4\alpha_8-274\alpha_0^3\alpha_8^2-408\alpha_0^2\alpha_1\alpha_8^2-170\alpha_0\alpha_1^2\alpha_8^2-32\alpha_1^3\alpha_8^2-306\alpha_0^2\alpha_2\alpha_8^2-$
$255\alpha_0\alpha_1\alpha_2\alpha_8^2-72\alpha_1^2\alpha_2\alpha_8^2-93\alpha_0\alpha_2^2\alpha_8^2-51\alpha_1\alpha_2^2\alpha_8^2-11\alpha_2^3\alpha_8^2-204\alpha_0^2\alpha_3\alpha_8^2-170\alpha_0\alpha_1\alpha_3\alpha_8^2-48\alpha_1^2\alpha_3\alpha_8^2-$
$124\alpha_0\alpha_2\alpha_3\alpha_8^2-68\alpha_1\alpha_2\alpha_3\alpha_8^2-22\alpha_2^2\alpha_3\alpha_8^2-39\alpha_0\alpha_3^2\alpha_8^2-20\alpha_1\alpha_3^2\alpha_8^2-13\alpha_2\alpha_3^2\alpha_8^2-2\alpha_3^3\alpha_8^2-102\alpha_0^2\alpha_4\alpha_8^2-$
$85\alpha_0\alpha_1\alpha_4\alpha_8^2-24\alpha_1^2\alpha_4\alpha_8^2-62\alpha_0\alpha_2\alpha_4\alpha_8^2-34\alpha_1\alpha_2\alpha_4\alpha_8^2-11\alpha_2^2\alpha_4\alpha_8^2-39\alpha_0\alpha_3\alpha_4\alpha_8^2-20\alpha_1\alpha_3\alpha_4\alpha_8^2-13\alpha_2\alpha_3\alpha_4\alpha_8^2-$
$3\alpha_3^2\alpha_4\alpha_8^2-8\alpha_0\alpha_4^2\alpha_8^2-3\alpha_1\alpha_4^2\alpha_8^2-2\alpha_2\alpha_4^2\alpha_8^2-\alpha_3\alpha_4^2\,\alpha_8^2-432\alpha_0^2\alpha_6\alpha_8^2-416\alpha_0\alpha_1\alpha_6\alpha_8^2-100\alpha_1^2\alpha_6\alpha_8^2-$
$312\alpha_0\alpha_2\alpha_6\alpha_8^2-150\alpha_1\alpha_2\alpha_6\alpha_8^2-54\alpha_2^2\alpha_6\alpha_8^2-208\alpha_0\alpha_3\alpha_6\alpha_8^2-100\alpha_1\alpha_3\alpha_6\alpha_8^2-72\alpha_2\alpha_3\alpha_6\alpha_8^2-22\alpha_3^2\alpha_6\alpha_8^2-$
$104\alpha_0\alpha_4\alpha_6\alpha_8^2-50\alpha_1\alpha_4\alpha_6\alpha_8^2-36\alpha_2\alpha_4\alpha_6\alpha_8^2-22\alpha_3\alpha_4\alpha_6\alpha_8^2-4\alpha_4^2\alpha_6\alpha_8^2-215\alpha_0\alpha_6^2\alpha_8^2-100\alpha_1\alpha_6^2\alpha_8^2-75\alpha_2\alpha_6^2\alpha_8^2-$
$50\alpha_3\alpha_6^2\alpha_8^2-25\alpha_4\alpha_6^2\alpha_8^2-32\alpha_6^3\alpha_8^2-216\alpha_0^2\alpha_7\alpha_8^2-208\alpha_0\alpha_1\alpha_7\alpha_8^2-50\alpha_1^2\alpha_7\alpha_8^2-156\alpha_0\alpha_2\alpha_7\alpha_8^2-75\alpha_1\alpha_2\alpha_7\alpha_8^2-$
$27\alpha_2^2\alpha_7\alpha_8^2-104\alpha_0\alpha_3\alpha_7\alpha_8^2-50\alpha_1\alpha_3\alpha_7\alpha_8^2-36\alpha_2\alpha_3\alpha_7\alpha_8^2-11\alpha_3^2\alpha_7\alpha_8^2-52\alpha_0\alpha_4\alpha_7\alpha_8^2-25\alpha_1\alpha_4\alpha_7\alpha_8^2-$
$18\alpha_2\alpha_4\alpha_7\alpha_8^2-11\alpha_3\alpha_4\alpha_7\alpha_8^2-2\alpha_4^2\alpha_7\alpha_8^2-215\alpha_0\alpha_6\alpha_7\alpha_8^2-100\alpha_1\alpha_6\alpha_7\alpha_8^2-75\alpha_2\alpha_6\alpha_7\alpha_8^2-50\alpha_3\alpha_6\alpha_7\alpha_8^2-$
$25\alpha_4\alpha_6\alpha_7\alpha_8^2-48\alpha_6^2\alpha_7\alpha_8^2-57\alpha_0\alpha_7^2\alpha_8^2-28\alpha_1\alpha_7^2\alpha_8^2-21\alpha_2\alpha_7^2\alpha_8^2-14\alpha_3\alpha_7^2\alpha_8^2-7\alpha_4\alpha_7^2\alpha_8^2-26\alpha_6\alpha_7^2\alpha_8^2-$
$5\alpha_7^3\alpha_8^2-126\alpha_0^2\alpha_8^3-120\alpha_0\alpha_1\alpha_8^3-28\alpha_1^2\alpha_8^3-90\alpha_0\alpha_2\alpha_8^3-42\alpha_1\alpha_2\alpha_8^3-15\alpha_2^2\alpha_8^3-60\alpha_0\alpha_3\alpha_8^3-28\alpha_1\alpha_3\alpha_8^3-$
$20\alpha_2\alpha_3\alpha_8^3-6\alpha_3^2\alpha_8^3-30\alpha_0\alpha_4\alpha_8^3-14\alpha_1\alpha_4\alpha_8^3-10\alpha_2\alpha_4\alpha_8^3-6\alpha_3\alpha_4\alpha_8^3-\alpha_4^2\alpha_8^3-132\alpha_0\alpha_6\alpha_8^3-64\alpha_1\alpha_6\alpha_8^3-$
$48\alpha_2\alpha_6\alpha_8^3-32\alpha_3\alpha_6\alpha_8^3-16\alpha_4\alpha_6\alpha_8^3-33\alpha_6^2\alpha_8^3-66\alpha_0\alpha_7\alpha_8^3-32\alpha_1\alpha_7\alpha_8^3-24\alpha_2\alpha_7\alpha_8^3-16\alpha_3\alpha_7\alpha_8^3-8\alpha_4\alpha_7\alpha_8^3-$
$33\alpha_6\alpha_7\alpha_8^3-9\alpha_7^2\alpha_8^3-27\alpha_0\alpha_8^4-12\alpha_1\alpha_8^4-9\alpha_2\alpha_8^4-6\alpha_3\alpha_8^4-3\alpha_4\alpha_8^4-14\alpha_6\alpha_8^4-7\alpha_7\alpha_8^4-2\alpha_8^5)+q(72\alpha_0^5+$
$192\alpha_0^4\alpha_1+188\alpha_0^3\alpha_1^2+108\alpha_0^2\alpha_1^3+24\alpha_0\alpha_1^4-4\alpha_1^5+144\alpha_0^4\alpha_2+282\alpha_0^3\alpha_1\alpha_2+243\alpha_0^2\alpha_1^2\alpha_2+72\alpha_0\alpha_1^3\alpha_2-$
$15\alpha_1^4\alpha_2+102\alpha_0^3\alpha_2^2+171\alpha_0^2\alpha_1\alpha_2^2+78\alpha_0\alpha_1^2\alpha_2^2-21\alpha_1^3\alpha_2^2+36\alpha_0^2\alpha_2^3+36\alpha_0\alpha_1\alpha_2^3-13\alpha_1^2\alpha_2^3+6\alpha_0\alpha_2^4-3\alpha_1\alpha_2^4+$
$96\alpha_0^4\alpha_3+188\alpha_0^3\alpha_1\alpha_3+162\alpha_0^2\alpha_1^2\alpha_3+48\alpha_0\alpha_1^3\alpha_3-10\alpha_1^4\alpha_3+136\alpha_0^3\alpha_2\alpha_3+228\alpha_0^2\alpha_1\alpha_2\alpha_3+104\alpha_0\alpha_1^2\alpha_2\alpha_3-$
$28\alpha_1^3\alpha_2\alpha_3+72\alpha_0^2\alpha_2^2\alpha_3+72\alpha_0\alpha_1\alpha_2^2\alpha_3-26\alpha_1^2\alpha_2^2\alpha_3+16\alpha_0\alpha_2^3\alpha_3-8\alpha_1\alpha_2^3\alpha_3+42\alpha_0^3\alpha_3^2+66\alpha_0^2\alpha_1\alpha_3^2+32\alpha_0\alpha_1^2\alpha_3^2-$
$8\alpha_1^3\alpha_3^2+42\alpha_0^2\alpha_2\alpha_3^2+44\alpha_0\alpha_1\alpha_2\alpha_3^2-15\alpha_1^2\alpha_2\alpha_3^2+14\alpha_0\alpha_2^2\alpha_3^2-7\alpha_1\alpha_2^2\alpha_3^2+6\alpha_0^2\alpha_3^3+8\alpha_0\alpha_1\alpha_3^3-2\alpha_1^2\alpha_3^3+$
$4\alpha_0\alpha_2\alpha_3^3-2\alpha_1\alpha_2\alpha_3^3+48\alpha_0^4\alpha_4+94\alpha_0^3\alpha_1\alpha_4+81\alpha_0^2\alpha_1^2\alpha_4+24\alpha_0\alpha_1^3\alpha_4-5\alpha_1^4\alpha_4+68\alpha_0^3\alpha_2\alpha_4+114\alpha_0^2\alpha_1\alpha_2\alpha_4+$
$52\alpha_0\alpha_1^2\alpha_2\alpha_4-14\alpha_1^3\alpha_2\alpha_4+36\alpha_0^2\alpha_2^2\alpha_4+36\alpha_0\alpha_1\alpha_2^2\alpha_4-13\alpha_1^2\alpha_2^2\alpha_4+8\alpha_0\alpha_2^3\alpha_4-4\alpha_1\alpha_2^3\alpha_4+42\alpha_0^3\alpha_3\alpha_4+$
$66\alpha_0^2\alpha_1\alpha_3\alpha_4+32\alpha_0\alpha_1^2\alpha_3\alpha_4-8\alpha_1^3\alpha_3\alpha_4+42\alpha_0^2\alpha_2\alpha_3\alpha_4+44\alpha_0\alpha_1\alpha_2\alpha_3\alpha_4-15\alpha_1^2\alpha_2\alpha_3\alpha_4+14\alpha_0\alpha_2^2\alpha_3\alpha_4-$
$7\alpha_1\alpha_2^2\alpha_3\alpha_4+9\alpha_0^2\alpha_3^2\alpha_4+12\alpha_0\alpha_1\alpha_3^2\alpha_4-3\alpha_1^2\alpha_3^2\alpha_4+6\alpha_0\alpha_2\alpha_3^2\alpha_4-3\alpha_1\alpha_2\alpha_3^2\alpha_4+8\alpha_0^3\alpha_4^2+9\alpha_0^2\alpha_1\alpha_4^2+6\alpha_0\alpha_1^2\alpha_4^2-$

$$\alpha_1^3\alpha_4^2 \ + 6\alpha_0^2\alpha_2\alpha_4^2 + 8\alpha_0\alpha_1\alpha_2\alpha_4^2 - 2\alpha_1^2\alpha_2\alpha_4^2 + 2\alpha_0\alpha_2^2\alpha_4^2 - \alpha_1\alpha_2^2 \ \alpha_4^2 + 3\alpha_0^2\alpha_3\alpha_4^2 + 4\alpha_0\alpha_1\alpha_3\alpha_4^2 - \alpha_1^2 \ \alpha_3\alpha_4^2 +$$
$$2\alpha_0\alpha_2\alpha_3\alpha_4^2 - \alpha_1 \ \alpha_2\alpha_3\alpha_4^2 + 240\alpha_0^4\alpha_6 + 512\alpha_0^3\alpha_1\alpha_6 + 376\alpha_0^2\alpha_1^2\alpha_6 + 112\alpha_0\alpha_1^3\alpha_6 - 4\alpha_1^4\alpha_6 + 384\alpha_0^3\alpha_2\alpha_6 +$$
$$564\alpha_0^2\alpha_1\alpha_2\alpha_6+252\alpha_0\alpha_1^2\alpha_2\alpha_6-12\alpha_1^3\alpha_2\alpha_6+204\alpha_0^2\alpha_2^2\alpha_6+180\alpha_0\alpha_1\alpha_2^2\alpha_6-12\alpha_1^2\alpha_2^2\alpha_6+40\alpha_0\alpha_2^3\alpha_6-4\alpha_1\alpha_2^3\alpha_6+$$
$$256\alpha_0^3\alpha_3\alpha_6 + 376\alpha_0^2\alpha_1\alpha_3\alpha_6 + 168\alpha_0\alpha_1^2\alpha_3\alpha_6 - 8\alpha_1^3\alpha_3\alpha_6 + 272\alpha_0^2\alpha_2\alpha_3\alpha_6 + 240\alpha_0\alpha_1\alpha_2\alpha_3\alpha_6 - 16\alpha_1^2\alpha_2\alpha_3\alpha_6 +$$
$$80\alpha_0\alpha_2^2\alpha_3\alpha_6 - 8\alpha_1\alpha_2^2\alpha_3\alpha_6 + 84\alpha_0^2\alpha_3^2\alpha_6 + 72\alpha_0\alpha_1\alpha_3^2\alpha_6 - 4\alpha_1^2\alpha_3^2\alpha_6 + 48\alpha_0\alpha_2\alpha_3^2\alpha_6 - 4\alpha_1\alpha_2\alpha_3^2\alpha_6 + 8\alpha_0\alpha_3^3\alpha_6 +$$
$$128\alpha_0^3\alpha_4\alpha_6 + 188\alpha_0^2\alpha_1\alpha_4\alpha_6 + 84\alpha_0\alpha_1^2\alpha_4\alpha_6 - 4\alpha_1^3\alpha_4\alpha_6 + 136\alpha_0^2\alpha_2\alpha_4\alpha_6 + 120\alpha_0\alpha_1\alpha_2\alpha_4\alpha_6 - 8\alpha_1^2\alpha_2\alpha_4\alpha_6 +$$
$$40\alpha_0\alpha_2^2\alpha_4\alpha_6-4\alpha_1\alpha_2^2\alpha_4\alpha_6+84\alpha_0^2\alpha_3\alpha_4\alpha_6+72\alpha_0\alpha_1\alpha_3\alpha_4\alpha_6-4\alpha_1^2\alpha_3\alpha_4\alpha_6+48\alpha_0\alpha_2\alpha_3\alpha_4\alpha_6-4\alpha_1\alpha_2\alpha_3\alpha_4\alpha_6+$$
$$12\alpha_0\alpha_3^2\alpha_4\alpha_6+16\alpha_0^2\alpha_4^2\alpha_6+12\alpha_0\alpha_1\alpha_4^2\alpha_6+8\alpha_0\alpha_2\alpha_4^2\alpha_6+4\alpha_0\alpha_3\alpha_4^2\alpha_6+314\alpha_0^3\alpha_6^2+496\alpha_0^2\alpha_1\alpha_6^2+232\alpha_0\alpha_1^2\alpha_6^2+$$
$$20\alpha_1^3\alpha_6^2 + 372\alpha_0^2\alpha_2\alpha_6^2 + 348\alpha_0\alpha_1\alpha_2\alpha_6^2 + 45\alpha_1^2\alpha_2\alpha_6^2 + 126\alpha_0\alpha_2^2\alpha_6^2 + 33\alpha_1\alpha_2^2\alpha_6^2 + 8\alpha_2^3\alpha_6^2 + 248\alpha_0^2\alpha_3\alpha_6^2 +$$
$$232\alpha_0\alpha_1\alpha_3\alpha_6^2+30\alpha_1^2\alpha_3\alpha_6^2+168\alpha_0\alpha_2\alpha_3\alpha_6^2+44\alpha_1\alpha_2\alpha_3\alpha_6^2+16\alpha_2^2\alpha_3\alpha_6^2+52\alpha_0\alpha_3^2\alpha_6^2+14\alpha_1\alpha_3^2\alpha_6^2+10\alpha_2\alpha_3^2\alpha_6^2+$$
$$2\alpha_3^3\alpha_6^2 + 124\alpha_0^2\alpha_4\alpha_6^2 + 116\alpha_0\alpha_1\alpha_4\alpha_6^2 + 15\alpha_1^2\alpha_4\alpha_6^2 + 84\alpha_0\alpha_2\alpha_4\alpha_6^2 + 22\alpha_1\alpha_2\alpha_4\alpha_6^2 + 8\alpha_2^2\alpha_4\alpha_6^2 + 52\alpha_0\alpha_3\alpha_4\alpha_6^2 +$$
$$14\alpha_1\alpha_3\alpha_4\alpha_6^2+10\alpha_2\alpha_3\alpha_4\alpha_6^2+3\alpha_3^2\alpha_4\alpha_6^2+10\alpha_0\alpha_4^2\alpha_6^2+3\alpha_1\alpha_4^2\alpha_6^2+2\alpha_2\alpha_4^2\alpha_6^2+\alpha_3\alpha_4^2 \ \alpha_6^2+202\alpha_0^2\alpha_6^3+208\alpha_0\alpha_1\alpha_6^3+$$
$$44\alpha_1^2\alpha_6^3 + 156\alpha_0\alpha_2\alpha_6^3 + 66\alpha_1\alpha_2\alpha_6^3 + 24\alpha_2^2\alpha_6^3 + 104\alpha_0\alpha_3\alpha_6^3 + 44\alpha_1\alpha_3\alpha_6^3 + 32\alpha_2\alpha_3\alpha_6^3 + 10\alpha_3^2\alpha_6^3 + 52\alpha_0\alpha_4\alpha_6^3 +$$
$$22\alpha_1\alpha_4\alpha_6^3+16\alpha_2\alpha_4\alpha_6^3+10\alpha_3\alpha_4\alpha_6^3+2\alpha_4^2\alpha_6^3+64\alpha_0\alpha_6^4+32\alpha_1\alpha_6^4+24\alpha_2\alpha_6^4+16\alpha_3\alpha_6^4+8\alpha_4\alpha_6^4+8\alpha_6^5+120\alpha_0^4\alpha_7+$$
$$256\alpha_0^3\alpha_1\alpha_7 + 188\alpha_0^2\alpha_1^2\alpha_7 + 56\alpha_0\alpha_1^3\alpha_7 - 2\alpha_1^4\alpha_7 + 192\alpha_0^3\alpha_2\alpha_7 + 282\alpha_0^2\alpha_1\alpha_2\alpha_7 + 126\alpha_0\alpha_1^2\alpha_2\alpha_7 - 6\alpha_1^3\alpha_2\alpha_7 +$$
$$102\alpha_0^2\alpha_2^2\alpha_7+90\alpha_0\alpha_1\alpha_2^2\alpha_7-6\alpha_1^2\alpha_2^2\alpha_7+20\alpha_0\alpha_2^3\alpha_7-2\alpha_1\alpha_2^3\alpha_7+128\alpha_0^3\alpha_3\alpha_7+188\alpha_0^2\alpha_1\alpha_3\alpha_7+84\alpha_0\alpha_1^2\alpha_3\alpha_7-$$
$$4\alpha_1^3\alpha_3\alpha_7 + 136\alpha_0^2\alpha_2\alpha_3\alpha_7 + 120\alpha_0\alpha_1\alpha_2\alpha_3\alpha_7 - 8\alpha_1^2\alpha_2\alpha_3\alpha_7 + 40\alpha_0\alpha_2^2\alpha_3\alpha_7 - 4\alpha_1\alpha_2^2\alpha_3\alpha_7 + 42\alpha_0^2\alpha_3^2\alpha_7 +$$
$$36\alpha_0\alpha_1\alpha_3^2\alpha_7-2\alpha_1^2\alpha_3^2\alpha_7+24\alpha_0\alpha_2\alpha_3^2\alpha_7-2\alpha_1\alpha_2\alpha_3^2\alpha_7+4\alpha_0\alpha_3^3\alpha_7+64\alpha_0^3\alpha_4\alpha_7+94\alpha_0^2\alpha_1\alpha_4\alpha_7+42\alpha_0\alpha_1^2\alpha_4\alpha_7-$$
$$2\alpha_1^3\alpha_4\alpha_7 + 68\alpha_0^2\alpha_2\alpha_4\alpha_7 + 60\alpha_0\alpha_1\alpha_2\alpha_4\alpha_7 - 4\alpha_1^2\alpha_2\alpha_4\alpha_7 + 20\alpha_0\alpha_2^2\alpha_4\alpha_7 - 2\alpha_1\alpha_2^2\alpha_4\alpha_7 + 42\alpha_0^2\alpha_3\alpha_4\alpha_7 +$$
$$36\alpha_0\alpha_1\alpha_3\alpha_4\alpha_7 - 2\alpha_1^2\alpha_3\alpha_4\alpha_7 + 24\alpha_0\alpha_2\alpha_3\alpha_4\alpha_7 - 2\alpha_1\alpha_2\alpha_3\alpha_4\alpha_7 + 6\alpha_0\alpha_3^2\alpha_4\alpha_7 + 8\alpha_0^2\alpha_4^2\alpha_7 + 6\alpha_0\alpha_1\alpha_4^2\alpha_7 +$$
$$4\alpha_0\alpha_2\alpha_4^2\alpha_7 + 2\alpha_0\alpha_3\alpha_4^2\alpha_7 + 314\alpha_0^3\alpha_6\alpha_7 + 496\alpha_0^2\alpha_1\alpha_6\alpha_7 + 232\alpha_0\alpha_1^2\alpha_6\alpha_7 + 20\alpha_1^3\alpha_6\alpha_7 + 372\alpha_0^2\alpha_2\alpha_6\alpha_7 +$$
$$348\alpha_0\alpha_1\alpha_2\alpha_6\alpha_7+45\alpha_1^2\alpha_2\alpha_6\alpha_7+126\alpha_0\alpha_2^2\alpha_6\alpha_7+33\alpha_1\alpha_2^2\alpha_6\alpha_7+8\alpha_2^3\alpha_6\alpha_7+248\alpha_0^2\alpha_3\alpha_6\alpha_7+232\alpha_0\alpha_1\alpha_3\alpha_6\alpha_7+$$
$$30\alpha_1^2\alpha_3\alpha_6\alpha_7+168\alpha_0\alpha_2\alpha_3\alpha_6\alpha_7+44\alpha_1\alpha_2\alpha_3\alpha_6\alpha_7+16\alpha_2^2\alpha_3\alpha_6\alpha_7+52\alpha_0\alpha_3^2\alpha_6\alpha_7+14\alpha_1\alpha_3^2\alpha_6\alpha_7+10\alpha_2\alpha_3^2\alpha_6\alpha_7+$$
$$2\alpha_3^3\alpha_6\alpha_7+124\alpha_0^2\alpha_4\alpha_6\alpha_7+116\alpha_0\alpha_1\alpha_4\alpha_6\alpha_7+15\alpha_1^2\alpha_4\alpha_6\alpha_7+84\alpha_0\alpha_2\alpha_4\alpha_6\alpha_7+22\alpha_1\alpha_2\alpha_4\alpha_6\alpha_7+8\alpha_2^2\alpha_4\alpha_6\alpha_7+$$
$$52\alpha_0\alpha_3\alpha_4\alpha_6\alpha_7+14\alpha_1\alpha_3\alpha_4\alpha_6\alpha_7+10\alpha_2\alpha_3\alpha_4\alpha_6\alpha_7+3\alpha_3^2\alpha_4\alpha_6\alpha_7+10\alpha_0\alpha_4^2\alpha_6\alpha_7+3\alpha_1\alpha_4^2\alpha_6\alpha_7+2\alpha_2\alpha_4^2\alpha_6\alpha_7+$$
$$\alpha_3 \ \alpha_4^2\alpha_6\alpha_7+303\alpha_0^2\alpha_6^2\alpha_7+312\alpha_0\alpha_1\alpha_6^2\alpha_7+66\alpha_1^2\alpha_6^2\alpha_7+234\alpha_0\alpha_2\alpha_6^2\alpha_7+99\alpha_1\alpha_2\alpha_6^2\alpha_7+36\alpha_2^2\alpha_6^2\alpha_7+156\alpha_0\alpha_3\alpha_6^2\alpha_7+$$
$$66\alpha_1\alpha_3\alpha_6^2\alpha_7 + 48\alpha_2\alpha_3\alpha_6^2\alpha_7 + 15\alpha_3^2\alpha_6^2\alpha_7 + 78\alpha_0\alpha_4\alpha_6^2\alpha_7 + 33\alpha_1\alpha_4\alpha_6^2\alpha_7 + 24\alpha_2\alpha_4\alpha_6^2\alpha_7 + 15\alpha_3\alpha_4\alpha_6^2\alpha_7 +$$
$$3\alpha_4^2\alpha_6^2\alpha_7+128\alpha_0\alpha_6^3\alpha_7+64\alpha_1\alpha_6^3\alpha_7+48\alpha_2\alpha_6^3\alpha_7+32\alpha_3\alpha_6^3\alpha_7+16\alpha_4\alpha_6^3\alpha_7+20\alpha_6^4\alpha_7+74\alpha_0^3\alpha_7^2+112\alpha_0^2\alpha_1\alpha_7^2+$$
$$44\alpha_0\alpha_1^2\alpha_7^2+84\alpha_0^2\alpha_2\alpha_7^2+66\alpha_0\alpha_1\alpha_2\alpha_7^2+24\alpha_0\alpha_2^2\alpha_7^2+56\alpha_0^2\alpha_3\alpha_7^2+44\alpha_0\alpha_1\alpha_3\alpha_7^2+32\alpha_0\alpha_2\alpha_3\alpha_7^2+10\alpha_0\alpha_3^2\alpha_7^2+$$
$$28\alpha_0^2\alpha_4\alpha_7^2+22\alpha_0\alpha_1\alpha_4\alpha_7^2+16\alpha_0\alpha_2\alpha_4\alpha_7^2+10\alpha_0\alpha_3\alpha_4\alpha_7^2+2\alpha_0\alpha_4^2\alpha_7^2+141\alpha_0^2\alpha_6\alpha_7^2+136\alpha_0\alpha_1\alpha_6\alpha_7^2+22\alpha_1^2\alpha_6\alpha_7^2+$$
$$102\alpha_0\alpha_2\alpha_6\alpha_7^2+33\alpha_1\alpha_2\alpha_6\alpha_7^2+12\alpha_2^2\alpha_6\alpha_7^2+68\alpha_0\alpha_3\alpha_6\alpha_7^2+22\alpha_1\alpha_3\alpha_6\alpha_7^2+16\alpha_2\alpha_3\alpha_6\alpha_7^2+5\alpha_3^2\alpha_6\alpha_7^2+34\alpha_0\alpha_4\alpha_6\alpha_7^2+$$
$$11\alpha_1\alpha_4\alpha_6\alpha_7^2 + 8\alpha_2\alpha_4\alpha_6\alpha_7^2 + 5\alpha_3\alpha_4\alpha_6\alpha_7^2 + \alpha_4^2 \ \alpha_6\alpha_7^2 + 88\alpha_0\alpha_6^2\alpha_7^2 + 40\alpha_1\alpha_6^2\alpha_7^2 + 30\alpha_2\alpha_6^2\alpha_7^2 + 20\alpha_3\alpha_6^2\alpha_7^2 +$$
$$10\alpha_4\alpha_6^2\alpha_7^2 + 18\alpha_6^3\alpha_7^2 + 20\alpha_0^2\alpha_7^3 + 16\alpha_0\alpha_1\alpha_7^3 + 12\alpha_0\alpha_2\alpha_7^3 + 8\alpha_0\alpha_3\alpha_7^3 + 4\alpha_0\alpha_4\alpha_7^3 + 24\alpha_0\alpha_6\alpha_7^3 + 8\alpha_1\alpha_6\alpha_7^3 +$$
$$6\alpha_2\alpha_6\alpha_7^3+4\alpha_3\alpha_6\alpha_7^3+2\alpha_4\alpha_6\alpha_7^3+7\alpha_6^2\alpha_7^3+2\alpha_0\alpha_7^4+\alpha_6\alpha_7^4+180\alpha_0^4\alpha_8+384\alpha_0^3\alpha_1\alpha_8+282\alpha_0^2\alpha_1^2\alpha_8+108\alpha_0\alpha_1^3\alpha_8+$$
$$12\alpha_1^4\alpha_8+288\alpha_0^3\alpha_2\alpha_8+423\alpha_0^2\alpha_1\alpha_2\alpha_8+243\alpha_0\alpha_1^2\alpha_2\alpha_8+36\alpha_1^3\alpha_2\alpha_8+153\alpha_0^2\alpha_2^2\alpha_8+171\alpha_0\alpha_1\alpha_2^2\alpha_8+39\alpha_1^2\alpha_2^2\alpha_8+$$
$$36\alpha_0\alpha_2^3\alpha_8+18\alpha_1\alpha_2^3\alpha_8+3\alpha_2^4\alpha_8+192\alpha_0^3\alpha_3\alpha_8+282\alpha_0^2\alpha_1\alpha_3\alpha_8+162\alpha_0\alpha_1^2\alpha_3\alpha_8+24\alpha_1^3\alpha_3\alpha_8+204\alpha_0^2\alpha_2\alpha_3\alpha_8+$$
$$228\alpha_0\alpha_1\alpha_2\alpha_3\alpha_8 + 52\alpha_1^2\alpha_2\alpha_3\alpha_8 + 72\alpha_0\alpha_2^2\alpha_3\alpha_8 + 36\alpha_1\alpha_2^2\alpha_3\alpha_8 + 8\alpha_2^3\alpha_3\alpha_8 + 63\alpha_0^2\alpha_3^2\alpha_8 + 66\alpha_0\alpha_1\alpha_3^2\alpha_8 +$$
$$16\alpha_1^2\alpha_3^2\alpha_8 + 42\alpha_0\alpha_2\alpha_3^2\alpha_8 + 22\alpha_1\alpha_2\alpha_3^2\alpha_8 + 7\alpha_2^2\alpha_3^2\alpha_8 + 6\alpha_0\alpha_3^3\alpha_8 + 4\alpha_1\alpha_3^3\alpha_8 + 2\alpha_2\alpha_3^3\alpha_8 + 96\alpha_0^3\alpha_4\alpha_8 +$$
$$141\alpha_0^2\alpha_1\alpha_4\alpha_8+81\alpha_0\alpha_1^2\alpha_4\alpha_8+12\alpha_1^3\alpha_4\alpha_8+102\alpha_0^2\alpha_2\alpha_4\alpha_8+114\alpha_0\alpha_1\alpha_2\alpha_4\alpha_8+26\alpha_1^2\alpha_2\alpha_4\alpha_8+36\alpha_0\alpha_2^2\alpha_4\alpha_8+$$
$$18\alpha_1\alpha_2^2\alpha_4\alpha_8+4\alpha_2^3\alpha_4\alpha_8+63\alpha_0^2\alpha_3\alpha_4\alpha_8+66\alpha_0\alpha_1\alpha_3\alpha_4\alpha_8+16\alpha_1^2\alpha_3\alpha_4\alpha_8+42\alpha_0\alpha_2\alpha_3\alpha_4\alpha_8+22\alpha_1\alpha_2\alpha_3\alpha_4\alpha_8+$$
$$7\alpha_2^2\alpha_3\alpha_4\alpha_8+9\alpha_0\alpha_3^2\alpha_4\alpha_8+6\alpha_1\alpha_3^2\alpha_4\alpha_8+3\alpha_2\alpha_3^2\alpha_4\alpha_8+12\alpha_0^2\alpha_4^2\alpha_8+9\alpha_0\alpha_1\alpha_4^2\alpha_8+3\alpha_1^2\alpha_4^2\alpha_8+6\alpha_0\alpha_2\alpha_4^2\alpha_8+$$
$$4\alpha_1\alpha_2\alpha_4^2\alpha_8+\alpha_2^2 \ \alpha_4^2\alpha_8+3\alpha_0\alpha_3\alpha_4^2\alpha_8+2\alpha_1\alpha_3\alpha_4^2\alpha_8+\alpha_2 \ \alpha_3\alpha_4^2\alpha_8+480\alpha_0^3\alpha_6\alpha_8+768\alpha_0^2\alpha_1\alpha_6\alpha_8+376\alpha_0\alpha_1^2\alpha_6\alpha_8+$$
$$56\alpha_1^3\alpha_6\alpha_8+576\alpha_0^2\alpha_2\alpha_6\alpha_8+564\alpha_0\alpha_1\alpha_2\alpha_6\alpha_8+126\alpha_1^2\alpha_2\alpha_6\alpha_8+204\alpha_0\alpha_2^2\alpha_6\alpha_8+90\alpha_1\alpha_2^2\alpha_6\alpha_8+20\alpha_2^3\alpha_6\alpha_8+$$
$$384\alpha_0^2\alpha_3\alpha_6\alpha_8 + 376\alpha_0\alpha_1\alpha_3\alpha_6\alpha_8 + 84\alpha_1^2\alpha_3\alpha_6\alpha_8 + 272\alpha_0\alpha_2\alpha_3\alpha_6\alpha_8 + 120\alpha_1\alpha_2\alpha_3\alpha_6\alpha_8 + 40\alpha_2^2\alpha_3\alpha_6\alpha_8 +$$
$$84\alpha_0\alpha_3^2\alpha_6\alpha_8+36\alpha_1\alpha_3^2\alpha_6\alpha_8+24\alpha_2\alpha_3^2\alpha_6\alpha_8+4\alpha_3^3\alpha_6\alpha_8+192\alpha_0^2\alpha_4\alpha_6\alpha_8+188\alpha_0\alpha_1\alpha_4\alpha_6\alpha_8+42\alpha_1^2\alpha_4\alpha_6\alpha_8+$$
$$136\alpha_0\alpha_2\alpha_4\alpha_6\alpha_8 + 60\alpha_1\alpha_2\alpha_4\alpha_6\alpha_8 + 20\alpha_2^2\alpha_4\alpha_6\alpha_8 + 84\alpha_0\alpha_3\alpha_4\alpha_6\alpha_8 + 36\alpha_1\alpha_3\alpha_4\alpha_6\alpha_8 + 24\alpha_2\alpha_3\alpha_4\alpha_6\alpha_8 +$$
$$6\alpha_3^2\alpha_4\alpha_6\alpha_8 + 16\alpha_0\alpha_4^2\alpha_6\alpha_8 + 6\alpha_1\alpha_4^2\alpha_6\alpha_8 + 4\alpha_2\alpha_4^2\alpha_6\alpha_8 + 2\alpha_3\alpha_4^2\alpha_6\alpha_8 + 471\alpha_0^2\alpha_6^2\alpha_8 + 496\alpha_0\alpha_1\alpha_6^2\alpha_8 +$$
$$116\alpha_1^2\alpha_6^2\alpha_8+372\alpha_0\alpha_2\alpha_6^2\alpha_8+174\alpha_1\alpha_2\alpha_6^2\alpha_8+63\alpha_2^2\alpha_6^2\alpha_8+248\alpha_0\alpha_3\alpha_6^2\alpha_8+116\alpha_1\alpha_3\alpha_6^2\alpha_8+84\alpha_2\alpha_3\alpha_6^2\alpha_8+$$
$$26\alpha_3^2\alpha_6^2\alpha_8+124\alpha_0\alpha_4\alpha_6^2\alpha_8+58\alpha_1\alpha_4\alpha_6^2\alpha_8+42\alpha_2\alpha_4\alpha_6^2\alpha_8+26\alpha_3\alpha_4\alpha_6^2\alpha_8+5\alpha_4^2\alpha_6^2\alpha_8+202\alpha_0\alpha_6^3\alpha_8+104\alpha_1\alpha_6^3\alpha_8+$$
$$78\alpha_2\alpha_6^3\alpha_8+52\alpha_3\alpha_6^3\alpha_8+26\alpha_4\alpha_6^3\alpha_8+32\alpha_6^4\alpha_8+240\alpha_0^3\alpha_7\alpha_8+384\alpha_0^2\alpha_1\alpha_7\alpha_8+188\alpha_0\alpha_1^2\alpha_7\alpha_8+28\alpha_1^3\alpha_7\alpha_8+$$
$$288\alpha_0^2\alpha_2\alpha_7\alpha_8+282\alpha_0\alpha_1\alpha_2\alpha_7\alpha_8+63\alpha_1^2\alpha_2\alpha_7\alpha_8+102\alpha_0\alpha_2^2\alpha_7\alpha_8+45\alpha_1\alpha_2^2\alpha_7\alpha_8+10\alpha_2^3\alpha_7\alpha_8+192\alpha_0^2\alpha_3\alpha_7\alpha_8+$$
$$188\alpha_0\alpha_1\alpha_3\alpha_7\alpha_8+42\alpha_1^2\alpha_3\alpha_7\alpha_8+136\alpha_0\alpha_2\alpha_3\alpha_7\alpha_8+60\alpha_1\alpha_2\alpha_3\alpha_7\alpha_8+20\alpha_2^2\alpha_3\alpha_7\alpha_8+42\alpha_0\alpha_3^2\alpha_7\alpha_8+18\alpha_1\alpha_3^2\alpha_7\alpha_8+$$
$$12\alpha_2\alpha_3^2\alpha_7\alpha_8+2\alpha_3^3\alpha_7\alpha_8+96\alpha_0^2\alpha_4\alpha_7\alpha_8+94\alpha_0\alpha_1\alpha_4\alpha_7\alpha_8+21\alpha_1^2\alpha_4\alpha_7\alpha_8+68\alpha_0\alpha_2\alpha_4\alpha_7\alpha_8+30\alpha_1\alpha_2\alpha_4\alpha_7\alpha_8+$$
$$10\alpha_2^2\alpha_4\alpha_7\alpha_8+42\alpha_0\alpha_3\alpha_4\alpha_7\alpha_8+18\alpha_1\alpha_3\alpha_4\alpha_7\alpha_8+12\alpha_2\alpha_3\alpha_4\alpha_7\alpha_8+3\alpha_3^2\alpha_4\alpha_7\alpha_8+8\alpha_0\alpha_4^2\alpha_7\alpha_8+3\alpha_1\alpha_4^2\alpha_7\alpha_8+$$

$2\alpha_2\alpha_4^2\alpha_7\alpha_8+\alpha_3\ \alpha_4^2\alpha_7\alpha_8+471\alpha_0^2\alpha_6\alpha_7\alpha_8+496\alpha_0\alpha_1\alpha_6\alpha_7\alpha_8+116\alpha_1^2\alpha_6\alpha_7\alpha_8+372\alpha_0\alpha_2\alpha_6\alpha_7\alpha_8+174\alpha_1\alpha_2\alpha_6\alpha_7\alpha_8+$
$63\alpha_2^2\alpha_6\alpha_7\alpha_8+248\alpha_0\alpha_3\alpha_6\alpha_7\alpha_8+116\alpha_1\alpha_3\alpha_6\alpha_7\alpha_8+84\alpha_2\alpha_3\alpha_6\alpha_7\alpha_8+26\alpha_3^2\alpha_6\alpha_7\alpha_8+124\alpha_0\alpha_4\alpha_6\alpha_7\alpha_8+$
$58\alpha_1\alpha_4\alpha_6\alpha_7\alpha_8+42\alpha_2\alpha_4\alpha_6\alpha_7\alpha_8+26\alpha_3\alpha_4\alpha_6\alpha_7\alpha_8+5\alpha_4^2\alpha_6\alpha_7\alpha_8+303\alpha_0\alpha_6^2\alpha_7\alpha_8+156\alpha_1\alpha_6^2\alpha_7\alpha_8+117\alpha_2\alpha_6^2\alpha_7\alpha_8+$
$78\alpha_3\alpha_6^2\alpha_7\alpha_8+39\alpha_4\alpha_6^2\alpha_7\alpha_8+64\alpha_6^3\alpha_7\alpha_8+111\alpha_0^2\alpha_7^2\alpha_8+112\alpha_0\alpha_1\alpha_7^2\alpha_8+22\alpha_1^2\alpha_7^2\alpha_8+84\alpha_0\alpha_2\alpha_7^2\alpha_8+33\alpha_1\alpha_2\alpha_7^2\alpha_8+$
$12\alpha_2^2\alpha_7^2\alpha_8+56\alpha_0\alpha_3\alpha_7^2\alpha_8+22\alpha_1\alpha_3\alpha_7^2\alpha_8+16\alpha_2\alpha_3\alpha_7^2\alpha_8+5\alpha_3^2\alpha_7^2\alpha_8+28\alpha_0\alpha_4\alpha_7^2\alpha_8+11\alpha_1\alpha_4\alpha_7^2\alpha_8+8\alpha_2\alpha_4\alpha_7^2\alpha_8+$
$5\alpha_3\alpha_4\alpha_7^2\alpha_8+\alpha_4^2\ \alpha_7^2\alpha_8+141\alpha_0\alpha_6\alpha_7^2\alpha_8+68\alpha_1\alpha_6\alpha_7^2\alpha_8+51\alpha_2\alpha_6\alpha_7^2\alpha_8+34\alpha_3\alpha_6\alpha_7^2\alpha_8+17\alpha_4\alpha_6\alpha_7^2\alpha_8+$
$44\alpha_6^2\alpha_7^2\alpha_8+20\alpha_0\alpha_7^3\alpha_8+8\alpha_1\alpha_7^3\alpha_8+6\alpha_2\alpha_7^3\alpha_8+4\alpha_3\alpha_7^3\alpha_8+2\alpha_4\alpha_7^3\alpha_8+12\alpha_6\alpha_7^3\alpha_8+\alpha_7^4\alpha_8+178\alpha_0^3\alpha_8^2+$
$288\alpha_0^2\alpha_1\alpha_8^2+150\alpha_0\alpha_1^2\alpha_8^2+32\alpha_1^3\alpha_8^2+216\alpha_0^2\alpha_2\alpha_8^2+225\alpha_0\alpha_1\alpha_2\alpha_8^2+72\alpha_1^2\alpha_2\alpha_8^2+81\alpha_0\alpha_2^2\alpha_8^2+51\alpha_1\alpha_2^2\alpha_8^2+$
$11\alpha_2^3\alpha_8^2+144\alpha_0^2\alpha_3\alpha_8^2+150\alpha_0\alpha_1\alpha_3\alpha_8^2+48\alpha_1^2\alpha_3\alpha_8^2+108\alpha_0\alpha_2\alpha_3\alpha_8^2+68\alpha_1\alpha_2\alpha_3\alpha_8^2+22\alpha_2^2\alpha_3\alpha_8^2+33\alpha_0\alpha_3^2\alpha_8^2+$
$20\alpha_1\alpha_3^2\alpha_8^2+13\alpha_2\alpha_3^2\alpha_8^2+2\alpha_3^3\alpha_8^2+72\alpha_0^2\alpha_4\alpha_8^2+75\alpha_0\alpha_1\alpha_4\alpha_8^2+24\alpha_1^2\alpha_4\alpha_8^2+54\alpha_0\alpha_2\alpha_4\alpha_8^2+34\alpha_1\alpha_2\alpha_4\alpha_8^2+$
$11\alpha_2^2\alpha_4\alpha_8^2+33\alpha_0\alpha_3\alpha_4\alpha_8^2+20\alpha_1\alpha_3\alpha_4\alpha_8^2+13\alpha_2\alpha_3\alpha_4\alpha_8^2+3\alpha_3^2\alpha_4\alpha_8^2+6\alpha_0\alpha_4^2\alpha_8^2+3\alpha_1\alpha_4^2\alpha_8^2+2\alpha_2\alpha_4^2\alpha_8^2+$
$\alpha_3\alpha_4^2\ \alpha_8^2+356\alpha_0^2\alpha_6\alpha_8^2+384\alpha_0\alpha_1\alpha_6\alpha_8^2+100\alpha_1^2\alpha_6\alpha_8^2+288\alpha_0\alpha_2\alpha_6\alpha_8^2+150\alpha_1\alpha_2\alpha_6\alpha_8^2+54\alpha_2^2\alpha_6\alpha_8^2+192\alpha_0\alpha_3\alpha_6\alpha_8^2+$
$100\alpha_1\alpha_3\alpha_6\alpha_8^2+72\alpha_2\alpha_3\alpha_6\alpha_8^2+22\alpha_3^2\alpha_6\alpha_8^2+96\alpha_0\alpha_4\alpha_6\alpha_8^2+50\alpha_1\alpha_4\alpha_6\alpha_8^2+36\alpha_2\alpha_4\alpha_6\alpha_8^2+22\alpha_3\alpha_4\alpha_6\alpha_8^2+$
$4\alpha_4^2\alpha_6\alpha_8^2+233\alpha_0\alpha_6^2\alpha_8^2+124\alpha_1\alpha_6^2\alpha_8^2+93\alpha_2\alpha_6^2\alpha_8^2+62\alpha_3\alpha_6^2\alpha_8^2+31\alpha_4\alpha_6^2\alpha_8^2+50\alpha_6^3\alpha_8^2+178\alpha_0^2\alpha_7\alpha_8^2+$
$192\alpha_0\alpha_1\alpha_7\alpha_8^2+50\alpha_1^2\alpha_7\alpha_8^2+144\alpha_0\alpha_2\alpha_7\alpha_8^2+75\alpha_1\alpha_2\alpha_7\alpha_8^2+27\alpha_2^2\alpha_7\alpha_8^2+96\alpha_0\alpha_3\alpha_7\alpha_8^2+50\alpha_1\alpha_3\alpha_7\alpha_8^2+$
$36\alpha_2\alpha_3\alpha_7\alpha_8^2+11\alpha_3^2\alpha_7\alpha_8^2+48\alpha_0\alpha_4\alpha_7\alpha_8^2+25\alpha_1\alpha_4\alpha_7\alpha_8^2+18\alpha_2\alpha_4\alpha_7\alpha_8^2+11\alpha_3\alpha_4\alpha_7\alpha_8^2+2\alpha_4^2\alpha_7\alpha_8^2+233\alpha_0\alpha_6\alpha_7\alpha_8^2+$
$124\alpha_1\alpha_6\alpha_7\alpha_8^2+93\alpha_2\alpha_6\alpha_7\alpha_8^2+62\alpha_3\alpha_6\alpha_7\alpha_8^2+31\alpha_4\alpha_6\alpha_7\alpha_8^2+75\alpha_6^2\alpha_7\alpha_8^2+55\alpha_0\alpha_7^2\alpha_8^2+28\alpha_1\alpha_7^2\alpha_8^2+21\alpha_2\alpha_7^2\alpha_8^2+$
$14\alpha_3\alpha_7^2\alpha_8^2+7\alpha_4\alpha_7^2\alpha_8^2+35\alpha_6\alpha_7^2\alpha_8^2+5\alpha_7^3\alpha_8^2+87\alpha_0^2\alpha_8^3+96\alpha_0\alpha_1\alpha_8^3+28\alpha_1^2\alpha_8^3+72\alpha_0\alpha_2\alpha_8^3+42\alpha_1\alpha_2\alpha_8^3+$
$15\alpha_2^2\alpha_8^3+48\alpha_0\alpha_3\alpha_8^3+28\alpha_1\alpha_3\alpha_8^3+20\alpha_2\alpha_3\alpha_8^3+6\alpha_3^2\alpha_8^3+24\alpha_0\alpha_4\alpha_8^3+14\alpha_1\alpha_4\alpha_8^3+10\alpha_2\alpha_4\alpha_8^3+6\alpha_3\alpha_4\alpha_8^3+$
$\alpha_4^2\alpha_8^3+116\alpha_0\alpha_6\alpha_8^3+64\alpha_1\alpha_6\alpha_8^3+48\alpha_2\alpha_6\alpha_8^3+32\alpha_3\alpha_6\alpha_8^3+16\alpha_4\alpha_6\alpha_8^3+38\alpha_6^2\alpha_8^3+58\alpha_0\alpha_7\alpha_8^3+32\alpha_1\alpha_7\alpha_8^3+$
$24\alpha_2\alpha_7\alpha_8^3+16\alpha_3\alpha_7\alpha_8^3+8\alpha_4\alpha_7\alpha_8^3+38\alpha_6\alpha_7\alpha_8^3+9\alpha_7^2\alpha_8^3+21\alpha_0\alpha_8^4+12\alpha_1\alpha_8^4+9\alpha_2\alpha_8^4+6\alpha_3\alpha_8^4+3\alpha_4\alpha_8^4+$
$14\alpha_6\alpha_8^4+7\alpha_7\alpha_8^4+2\alpha_8^5))$.
Here, the constant parameters α_i satisfy the relation:

$$6\alpha_0+5\alpha_1+4\alpha_2+3\alpha_3+2\alpha_4+\alpha_5+4\alpha_6+2\alpha_7+3\alpha_8=0. \tag{55}$$

The holomorphy conditions $(C2), (C3)$ are new. Theorem 11.1 can be checked by a direct calculation.

Proposition 11.1. *The Hamiltonian I is its first integral.*

Remark 11.1. *For the Hamiltonian system in each coordinate system (x_i, y_i) $(i = 0, 1, \ldots, 8)$ given by $(C2)$ and $(C3)$ in Theorem 11.1, by eliminating x_i or y_i, we obtain the second-order ordinary differential equation. However, its form is not normal* (cf. [8, 9]).

12 Symmetry

Theorem 12.1. *The system* (54) *admits the affine Weyl group symmetry of type $E_8^{(1)}$ as the group of its Bäcklund transformations whose generators s_i, $i = 0, 1, \ldots, 8$ are explicitly given as follows*: *with the notation* $(*) := (q, p, t; \alpha_0, \alpha_1, \ldots, \alpha_8)$,

$$\begin{aligned}
&s_0 : (*) \rightarrow \left(q+\frac{\alpha_0}{p}, p, t; -\alpha_0, \alpha_1+\alpha_0, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6+\alpha_0, \alpha_7, \alpha_8+\alpha_0\right),\\
&s_1 : (*) \rightarrow \left(q, p-\frac{\alpha_1}{q}, t; \alpha_0+\alpha_1, -\alpha_1, \alpha_2+\alpha_1, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8\right),\\
&s_2 : (*) \rightarrow (q, p, t; \alpha_0, \alpha_1+\alpha_2, -\alpha_2, \alpha_3+\alpha_2, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8),\\
&s_3 : (*) \rightarrow (q, p, t; \alpha_0, \alpha_1, \alpha_2+\alpha_3, -\alpha_3, \alpha_4+\alpha_3, \alpha_5, \alpha_6, \alpha_7, \alpha_8),\\
&s_4 : (*) \rightarrow (q, p, t; \alpha_0, \alpha_1, \alpha_2, \alpha_3+\alpha_4, -\alpha_4, \alpha_5+\alpha_4, \alpha_6, \alpha_7, \alpha_8),\\
&s_5 : (*) \rightarrow (q, p, t; \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4+\alpha_5, -\alpha_5, \alpha_6, \alpha_7, \alpha_8),\\
&s_6 : (*) \rightarrow \left(q, p-\frac{\alpha_6}{q-1}, t; \alpha_0+\alpha_6, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, -\alpha_6, \alpha_7+\alpha_6, \alpha_8\right),\\
&s_7 : (*) \rightarrow (q, p, t; \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6+\alpha_7, -\alpha_7, \alpha_8),\\
&s_8 : (*) \rightarrow (q, p, t; \alpha_0+\alpha_8, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, -\alpha_8).
\end{aligned}$$

Theorem 12.1 can be checked by a direct calculation.

13 Space of initial conditions

Theorem 13.1. *After a series of explicit blowing-ups at eleven points including the infinitely near points of Σ_2 and successive blowing-down along the (-1)-curves $D^{(0)'} \cong \mathbb{P}^1$, $D_1^{(1)} \cong \mathbb{P}^1$ and $D_\infty^{(1)} \cong \mathbb{P}^1$, we obtain the rational surface $\tilde{S}$ of the system* (54) *and a birational morphism $\varphi : \tilde{S} \cdots \to \Sigma_2$. Its canonical divisor $K_{\tilde{S}}$ of $\tilde{S}$ is given by*

$$K_{\tilde{S}} = -D_0^{(1)}, \quad (D_0^{(1)})^2 = -3, \ D_0^{(1)} \cong \mathbb{P}^1, \tag{56}$$

where the symbol $D^{(0)'}$ denotes the strict transform of $D^{(0)}$, $D_\nu^{(1)}$ denote the exceptional divisors and $-K_{\Sigma_2} = 2D^{(0)}$, $D^{(0)} \cong \mathbb{P}^1$, $(D^{(0)})^2 = 2$.

Theorem 13.2. *The space of initial conditions S of the system* (54) *is obtained by gluing ten copies of $\mathbb{C}^2$:*

$$\begin{aligned} S &= \tilde{S} - (-K_{\tilde{S}})_{red} \\ &= \mathbb{C}^2 \cup \bigcup_{i=0}^{8} U_j, \\ \mathbb{C}^2 &\ni (q,p), \quad U_j \cong \mathbb{C}^2 \ni (x_j, y_j) \ (j = 0, 1, \dots, 8) \end{aligned} \tag{57}$$

via the birational and symplectic transformations r_j (see Theorem 11.1).

Proof of Theorems 13.1 and 13.2.
By a direct calculation, we see that the system (54) has three accessible singular points $a_\nu^{(0)} \in D^{(0)}$ $(\nu = 0, 1, \infty)$:

$$\begin{aligned} a_\nu^{(0)} &= \{(z_2, w_2) = (\nu, 0)\} \in U_2 \cap D^{(0)} \ (\nu = 0, 1), \\ a_\infty^{(0)} &= \{(z_3, w_3) = (0, 0)\} \in U_3 \cap D^{(0)}. \end{aligned} \tag{58}$$

We perform blowing-ups in Σ_2 at $a_\nu^{(0)}$, and let $D_\nu^{(1)}$ be the exceptional curves of the blowing-ups at $a_\nu^{(0)}$ for $\nu = 0, 1, \infty$. We can take three coordinate systems (u_ν, v_ν) around the points at infinity of the exceptional curves $D_\nu^{(1)}$ $(\nu = 0, 1, \infty)$, where

$$\begin{aligned} (u_\nu, v_\nu) &= \left(\frac{z_2 - \nu}{w_2}, w_2 \right) \ (\nu = 0, 1), \\ (u_\infty, v_\infty) &= \left(\frac{z_3}{w_3}, w_3 \right). \end{aligned} \tag{59}$$

Note that $\{(u_\nu, v_\nu) | v_\nu = 0\} \subset D_\nu^{(1)}$ for $\nu = 0, 1, \infty$. By a direct calculation, we see that the system (54) has eight accessible singular points $a_\nu^{(1)}$ for $\nu = 1, 2, 3, 4, 5, 6, 7, 8$ in $D_\nu^{(1)} \cong \mathbb{P}^1$ $(\nu = 0, 1, \infty)$.

$$\begin{aligned} &a_1^{(1)} = \{(u_0, v_0) = (\alpha_1, 0)\} \in D_0^{(1)}, \quad a_2^{(1)} = \{(u_0, v_0) = (\alpha_1 + \alpha_2, 0)\} \in D_0^{(1)}, \\ &a_3^{(1)} = \{(u_0, v_0) = (\alpha_1 + \alpha_2 + \alpha_3, 0)\} \in D_0^{(1)}, \quad a_4^{(1)} = \{(u_0, v_0) = (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4, 0)\} \in D_0^{(1)}, \\ &a_5^{(1)} = \{(u_0, v_0) = (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5, 0)\} \in D_0^{(1)}, \quad a_6^{(1)} = \{(u_1, v_1) = (\alpha_6, 0)\} \in D_1^{(1)}, \\ &a_7^{(1)} = \{(u_1, v_1) = (\alpha_6 + \alpha_7, 0)\} \in D_1^{(1)}, \\ &a_8^{(1)} = \{(u_\infty, v_\infty) = (\alpha_8, 0)\} \in D_\infty^{(1)} \end{aligned} \tag{60}$$

Let us perform blowing-ups at $a_j^{(1)}$, and denote $D_j^{(2)}$ for the exceptional curves, respectively. We take

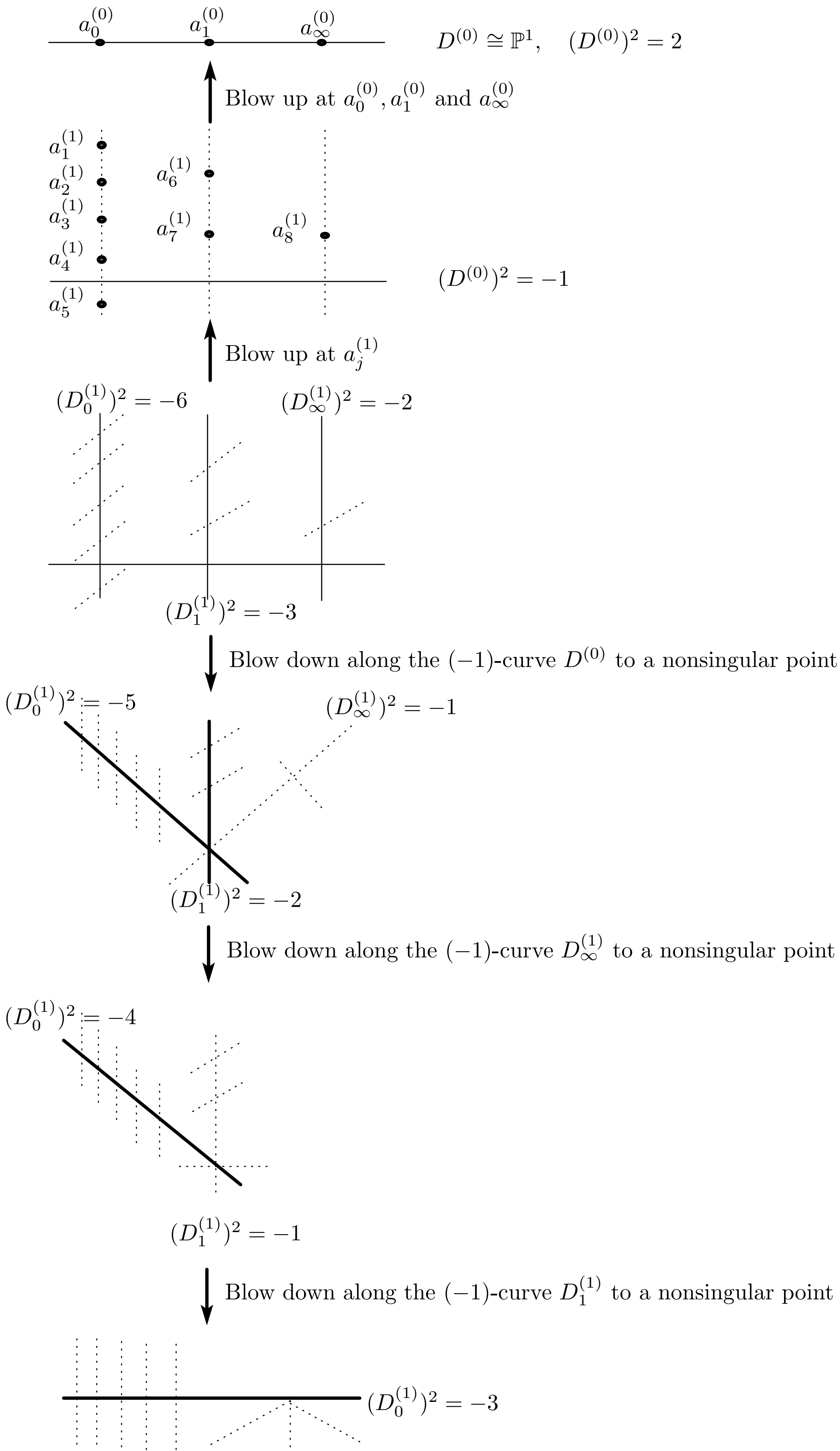


Figure 7: Resolution of accessible singular points

seven coordinate systems (W_j, V_j) around the points at infinity of $D_j^{(2)}$ for $j = 1,2,3,4,5,6,7,8$, where

$$\begin{aligned}
(W_1, V_1) &= \left(\frac{u_0 - \alpha_1}{v_0}, v_0\right), \\
(W_2, V_2) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2)}{v_0}, v_0\right), \\
(W_3, V_3) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2 + \alpha_3)}{v_0}, v_0\right), \\
(W_4, V_4) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)}{v_0}, v_1\right), \\
(W_5, V_5) &= \left(\frac{u_0 - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)}{v_0}, v_0\right), \\
(W_6, V_6) &= \left(\frac{u_1 - \alpha_6}{v_1}, v_1\right), \\
(W_7, V_7) &= \left(\frac{u_1 - (\alpha_6 + \alpha_7)}{v_1}, v_1\right), \\
(W_8, V_8) &= \left(\frac{u_\infty - \alpha_8}{v_\infty}, v_\infty\right).
\end{aligned} \tag{61}$$

For the strict transform of $D^{(0)}$, $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-ups, we also denote by same symbol, respectively. Here, the self-intersection number of $D^{(0)}, D_\nu^{(1)}$ is given by

$$(D^{(0)})^2 = -1. \quad (D_0^{(1)})^2 = -6, \quad (D_1^{(1)})^2 = -3, \quad (D_\infty^{(1)})^2 = -2. \tag{62}$$

In order to obtain a minimal compactification of the space of initial conditions, we must blow down along the (-1)-curves $D^{(0)} \cong \mathbb{P}^1$ to a nonsingular point. For the strict transform of $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-down, we also denote by same symbol, respectively. Here, the self-intersection number of $D_\nu^{(1)}$ is given by

$$(D_0^{(1)})^2 = -5, \quad (D_1^{(1)})^2 = -2, \quad (D_\infty^{(1)})^2 = -1. \tag{63}$$

We must blow down again along the (-1)-curve $D_\infty^{(1)} \cong \mathbb{P}^1$ to a nonsingular point. For the strict transform of $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-down, we also denote by same symbol, respectively. Here, the self-intersection number of $D_\nu^{(1)}$ is given by

$$(D_0^{(1)})^2 = -4, \quad (D_1^{(1)})^2 = -1. \tag{64}$$

We must blow down again along the (-1)-curve $D_1^{(1)} \cong \mathbb{P}^1$ to a nonsingular point. For the strict transform of $D_\nu^{(1)}$ and $D_j^{(2)}$ by the blowing-down, we also denote by same symbol, respectively. Here, the self-intersection number of $D_0^{(1)}$ is given by

$$(D_0^{(1)})^2 = -3. \tag{65}$$

Let $\tilde{S} \cdots \to \Sigma_2$ be the composition of above eleven times blowing-ups and three times blowing-downs. Then, we see that the canonical divisor class $K_{\tilde{S}}$ of $\tilde{S}$ is given by

$$K_{\tilde{S}} := -D_0^{(1)}, \tag{66}$$

where the self-intersection number of $D_0^{(1)} \cong \mathbb{P}^1$ is given by

$$(D_0^{(1)})^2 = -3. \tag{67}$$

We see that $\tilde{S}-(-K_{\tilde{S}})_{red}$ is covered by ten Zariski open sets

$$\begin{aligned}&\mathrm{Spec}\ \mathbb{C}[\mathrm{W_j},\mathrm{V_j}] \quad (\mathrm{j}=1,2,3,4,5,6,7,8),\\&\mathrm{Spec}\ \mathbb{C}[\mathrm{z_0},\mathrm{w_0}],\\&\mathrm{Spec}\ \mathbb{C}[\mathrm{z_1},\mathrm{w_1}].\end{aligned} \tag{68}$$

The relations between (W_j,V_j) and (x_j,y_j) are given by

$$(-W_j,V_j)=(x_j,y_j) \quad (j=1,2,3,4,5,6,7,8). \tag{69}$$

We see that the pole divisor of the symplectic 2-form $dp\wedge dq$ coincides with $(-K_{\tilde{S}})_{red}$. Thus, we have completed the proof of Theorems 13.1 and 13.2. □

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